Exploring the Effect of Wealth Distribution on Efficiency Using a Model of Land Tenancy with Limited Liability

Nicholas Reynolds

Abstract

When perfect information is present in a model, the initial distribution of wealth will have no effect on whether or not equilibrium is Pareto efficient. However, as Stiglitz (1987) emphasized, this no longer holds when imperfect information is introduced: the initial distribution of wealth can affect whether or not an economy is Pareto efficient. One reason, for this effect is that the distribution of wealth can effect whether or not moral hazard, and the inefficiencies associated with it, arise. In this paper, I demonstrate an example of this effect using first a model of land tenure with limited liability found in Ray and Singh (2001) and then a version of their model that is adapted by the addition of a second input into production. This adaption allows me to demonstrate, under certain conditions, that the distribution of wealth not only can affect whether or not the equilibrium allocation is first-best efficient but further whether it is even constrained efficient.

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1. Introduction

In neoclassical economic models with perfect information it is possible to prove that issues of equity and efficiency can be treated separately. Specifically, it can be shown that whether or not an economy is Pareto efficient does not depend on the initial distribution of wealth. Thus, efficiency concerns are dealt with by focusing on correcting market failures, and equity concerns are addressed by redistributing initial endowments and then letting the market mechanism operate without any worry about the efficiency of the economy. However, as work by Joseph Stiglitz and others in the economics of information has shown many of the basic conclusions of the traditional neoclassical model do not hold when imperfect information, such as asymmetric information between worker and capitalist, is introduced. For example, the Fundamental Theorems of Welfare Economics do not hold: in general markets with imperfect information are not Pareto efficient at equilibrium (Greenwald and Stiglitz, 1986). Further, the separability of equity and efficiency no longer holds in general in economies with imperfect information: whether or not an economy is Pareto efficient at equilibrium can depend on the initial distribution of wealth.

This non-separability arises for a number of related reasons. In this paper, I will be focusing on one specific reason. Asymmetric information between a lender and an investor, between a landlord and a farm laborer, or between an insurer and an individual being insured can lead to problems of moral hazard, which lead to inefficiencies. Importantly, the existence of the above transactions and the moral hazard and inefficiencies associated with them can depend on the distribution of wealth. Stiglitz (1987) writes, "The inefficiencies with which we have been concerned arise because of the asymmetric information between two parties—between landlord

and worker, or between worker and capitalist. But whether these transactions arise is, at least partly, determined by the distribution of wealth" (p. 16).

Sharecropping is a standard example used by Stiglitz and others to demonstrate this non-separability. For example, he writes "The incentive problems associated with sharecropping simply would not arise if all tenants owned the land upon which they worked" (p. 29). However, Stiglitz, and others do not appear to have formally modeled how distribution affects efficiency through its effect on the existence of moral hazard. There seem to be some difficulties with using sharecropping, specifically a model of sharecropping such as that presented by Stiglitz (1974) that focuses on risk-sharing, to formally model this effect. Thus in this paper I will use a model of land-tenancy with risk-neutral tenants and limited liability constraints, meaning a tenant can default if the payment is greater than their total wealth. In such a model, sharecropping does not develop but moral hazard arises for tenants of certain wealth levels (Ray and Singh, 2001).

Using the Ray and Singh model and then a slightly adapted version I will be able to demonstrate that the Pareto efficiency of the equilibrium allocation depends on the distribution of wealth. I first use an unadapted version of the Ray and Singh model and then adapt it slightly to obtain a stronger result, described below.

First, using a straight version of the Ray and Singh model I will demonstrate:

(1) that an equal distribution where all tenants have wealth above a certain cut-off level will lead to an equilibrium that is first best Pareto efficient.

(2) that an unequal distribution where some tenants have wealth above a certain level and

some have wealth below that level will lead to an equilibrium allocation that is not first-

best Pareto efficient, but is constrained Pareto efficient[1]

Then, using an adapted version of the Ray and Singh model that includes a second input

in production (which can be understood as fertilizer), I will demonstrate the stronger conclusion

that distribution can affect the *constrained* Pareto efficiency of equilibrium, under certain

conditions. Specifically, I will show given certain conditions:

(1) that an equal distribution where all tenants have wealth above a certain cutoff level will

lead to an equilibrium that is first-best Pareto efficient

(2) that an unequal distribution with some tenants below the cut-off wealth slevel (and some

above it) will lead to an equilibrium that is not even constrained Pareto efficient, a simple

tax-and-subsidy scheme can lead to a Pareto improvement

Overall, I will demonstrate that in the presence of imperfect information whether or not

an equilibrium allocation is constrained Pareto efficient can depend on the distribution of wealth.

This is due to the fact that the existence of situations with moral hazard can depend on the

distribution of wealth.

This paper will begin with a review of relevant literature, starting with literature on the

effect of introducing imperfect information into economic models, then proceeding to a more

specific summary of literature related to the effect of moral hazard in sharecropping and land

tenancy. Following this will be a presentation of the Ray and Singh model, unadapted and then

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[1] If an allocation is first-best efficient, even the introduction of perfect information would not represent a Pareto improvement. More weakly, if an allocation would be Pareto dominated by the perfect information equilibrium but there are no possible Pareto improvements taking the imperfect information as given than that allocation is constrained Pareto efficient. Finally, if there is a possible Pareto improvement to an allocation even taking the imperfect information into account, such as a tax-and-subsidy scheme, then the allocation is not even constrained Pareto efficient. The Greenwald and Stiglitz (1986) result described above demonstrated that with the presence of imperfect information the equilibrium allocation is not even *constrained* Pareto efficient under most circumstances.

with the addition of fertilizer, and a demonstration of the conclusions outlined above. Next, I will describe some caveats and possible areas for further inquiry. Finally, I will conclude by summarizing my findings and their implications.

2. Background and Literature Review

This paper draws primarily on literature in the fields of information economics and development economics. As described above, information economics deals in general with the effect that the introduction of imperfect information of various forms has on economic models and their conclusions. This paper focuses on one type of inefficiency that arises from imperfect information: moral hazard. Specifically, it deals with how the existence of moral hazard can depend on the distribution of wealth. This analysis is carried out with an example from an institution prevalent in developing countries: land tenure contracts. Thus, literature from development economics, specifically that focusing on the form of rural institutions and organizations, is relevant as well.

Information Economics

In combination the two fundamental theorems of welfare economics seem to provide an almost impenetrable argument for the superiority of the market system as a way to allocate an economy's resources. If one is concerned about the efficiency of the economy, the First Theorem tells us that the market mechanism will always lead to a Pareto efficient allocation in equilibrium, and therefore that no one can be made better off without making at least one person worse off. If one is still concerned with the equity of the allocation, whether the equilibrium allocation is fair or just, the Second Theorem tells us that any Pareto efficient allocation can be achieved through the market mechanism, all that needs to be changed are the initial endowments. In this traditional

story then concerns about efficiency and concerns about equity are separate. To ensure efficiency a government just needs to allow the market to work and deal with limited market failures. Separately, the government can make lump-sum transfers to obtain what society deems to be an equitable distribution.

This traditional story works very well as long as the very specific assumptions of the traditional neo-classical general equilibrium model hold. However, in many cases the fundamental conclusions fail to hold when even minor changes are introduced. The relaxation of the assumption of perfect information in particular leads many of the basic conclusions of the neo-classical model, such as the law of one price and the Fundamental Theorems of Welfare economics, to no longer hold (Stiglitz 1987).

Joseph Stiglitz, summarizing the contributions of information economics after its first decade of existence, writes:

> We have learned that much of what we believed before is of only limited validity; that the traditional competitive equilibrium analysis, though having the superficial appearance of generality—in terms of superscripts and subscripts—is indeed not very general; the theory is not robust to slight alterations in the informational assumptions. It is but a special—and not very plausible—'example' among the possible set of informational assumptions which can be employed to characterize an economy (Stiglitz 1985, p. 21).

For example, Greenwald and Stiglitz (1986) demonstrate that in general when imperfect information is present the market equilibrium will not be Pareto efficient, simple tax-and-subsidy schemes can in theory lead to Pareto improvements. This demonstrates not only that imperfect information leads to inefficiencies in comparison to a world of perfect information, that the equilibrium is not first-best efficient, but further that the equilibrium is not even Pareto efficient taking the information problems is given, it is not even constrained Pareto efficient. This

conclusion demonstrates that with the introduction of imperfect information the First Fundamental Theorem of Welfare Economics no longer holds in general.

Moral hazard is one of the most commonly studied ways that imperfect information can lead to inefficiencies. Also known as hidden action, it refers to the fact that after a contract is made, one party to the contract has an incentive to act in a way that will benefit themselves at the expense of the other party (Collins dictionary of economics, 2006). With perfect information the contract can include stipulations against such actions, but when information is imperfect one party may not be able to observe all of the other's actions and thus cannot enforce such stipulations. A common example of moral hazard involves insurance. For example, if an individual purchases fire insurance they will be more likely to take actions that increase the risk of fire because they will not bear (all) of the costs of a possible fire. Moral hazard can arise in a number of situations. For example, if worker has more information about their work effort than their employer, they will incentive to not work as hard as they would if they were working for themselves or if the employer had perfect information about their labor (Shapiro and Stiglitz, 1984).

(Non)-separability of Equity and Efficiency

Another conclusion of the perfect information model that fails to hold under more general informational assumptions is the idea that issues of equity and efficiency are separable. What exactly is meant by the separation of equity and efficiency, and thus what is meant if they cannot be separated, is not always clear or consistent in the literature even among the same authors. Often, it refers to the traditional story described above about the role of a theoretical government in an economy. With perfect information, the fundamental theorems of welfare economics demonstrate that concerns about efficiency and concerns about equity are completely separable

for a theoretical government. If a government is concerned that the particular Pareto efficient allocation that exists at equilibrium is not socially-desirable, it can institute lump-sum transfers to change the distribution of wealth. It can then allow the market to operate, confident in the fact that the equilibrium that results will be Pareto efficient.

Most simply, this story breaks down with the introduction of imperfect information because it is not feasible for a government to institute lump-sum transfers when information is not perfect. If it is assumed that a government does not have perfect information about individuals, it will not be able to costlessly determine what transfers it wishes to make. For example, if a government wishes to enact redistributions to make wealth or income distribution more equitable, it must design taxes that depend on wealth or income level. However, by designing taxes to depend on some variable under individuals' control, the government is distorting incentives. The redistributions are no longer lump-sum transfers; they are distortionary. Thus by making redistributions the government is now introducing distortions into the economy that could affect efficiency. Efficiency and equity cannot be separated because attempts at bringing about equitable or socially-desirable allocations will now have the potential to affect the efficiency of those allocations (Stiglitz, 1985).

However, even if lump-sum transfers were possible, equity and efficiency would not be separable with the presence of imperfect information. To understand why, it is useful to introduce a related but slightly different way of understanding the separability of equity and efficiency. Stiglitz (1985) writes, "One of the important consequences of the Fundamental Theorem of Welfare Economics was that it enabled a neat separation of efficiency and equity issues. In particular, whether the economy is or is not Pareto efficient did not depend on the distribution of wealth" (15). This is another way of understanding the separation of equity and

efficiency. If the Pareto efficiency of equilibrium depends on the distribution of wealth, a government's attempts at achieving equity through even lump-sum redistribution could lead to an equilibrium that is not Pareto efficient.

Stiglitz (1985) writes that with the introduction of imperfect information, whether or not an economy is Pareto efficient can depend on the distribution of wealth. He refers to two reasons that this can occur. First, in an economy with information problems that lead to inefficiencies, the distribution of wealth determines whether it is feasible to design Pareto improvements, thus it determines whether allocations are or are not constrained Pareto efficient. This result is demonstrated formally in Shapiro and Stiglitz (1984). In a model of the labor market with asymmetric information between workers and employers they demonstrate that the distribution of the ownership of capital affects whether or not Pareto improvements can be designed. Specifically, when it is assumed that ownership is distributed equally among all workers the equilibrium is not constrained Pareto efficient, a tax and subsidy scheme can be designed that leads to a Pareto improvement. However, when it is assumed that there are too distinct groups, owners and workers, it is not possible to design a Pareto improving tax and subsidy scheme, and thus the equilibrium is constrained Pareto efficient.

As Stiglitz explains, another reason that Pareto efficiency can depend on distribution is that the existence of inefficiencies associated with imperfect information, such as moral hazard, can depend on the distribution of wealth. If a worker has complete information about their work effort, but that information is costly to his employer (for example a landowner or capitalist) there will be added costs to the labor relations due to problems of moral hazard, either in the form of decreased work effort by the worker or added monitoring costs for the employer. As Stiglitz

emphasizes whether these transactions and the inefficiencies associated with them arise "is, at least partly determined by the distribution of wealth" (1985, p. 15).

In a number of articles Stiglitz uses the institution of sharecropping as an example. In Stiglitz (1991) he writes, "The incentive problems associated with sharecropping simply would not arise if all tenants owned the land upon which they worked" (p. 29). Similarly in Stiglitz (1985), he writes "The moral hazard problems associated with sharecropping arise partly from the fact that workers do not own the land upon which they work … More generally a basic insight of the New New welfare economics is that whether the economy is or is not Pareto efficient may depend on the initial distribution of wealth" (p. 31).

While Stiglitz frequently uses sharecropping as an example of the non-separability of efficiency and distribution, he has not demonstrated the effect formally. When describing the issue of equity and distribution he usually cites the result from Shapiro and Stiglitz (1984), which as described above is a very different example from the sharecropping example: focusing on the effect distribution has on the possibility of designing Pareto improvements rather than its effect on the existence of moral hazard inefficiencies.

Land Tenure with Limited Liability

While the effect of distribution on efficiency is frequently referenced by Stiglitz, it is usually as a side note, as part of the larger point that imperfect information changes many basic results of neo-classical models. In contrast, Hoff (1996) and Bardhan, Bowles and Gintis (1999) treat the effect of distribution on efficiency as the primary focus of their papers. Both present the idea of using a model of land tenure with limited-liability constraints to examine issues of equity and efficiency. In their models, which draw from earlier work by Shetty (1988), it is assumed that there is some randomness in output based on weather and other conditions. Further, it is

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assumed that there is an implied default clause in the contract, if the output is very bad and a tenant does not have enough wealth to cover the fixed rent the tenant will default. Thus moral hazard arises for tenants with wealth low enough to have a chance of default and these tenants put out less than efficient levels of labor.

The use of limited-liability constraint models makes a lot of sense, in contrast with Stiglitz's risk-sharing model of sharecropping, because such a model more naturally demonstrates the effect of distribution on the existence of incentive problems. Specifically, it is clear directly from the model that whether or not moral hazard arises depends on a tenant's wealth level. Thus it seems to follow that the efficiency of the economy as a whole could depend on the wealth distribution. However, Hoff and Bardhan, Bowles and Gintis do not focus their analysis on the effect of distribution on Pareto efficiency specifically. Hoff focuses on the effect that land redistribution could have on total output. Bardhan, Bowles and Gintis develop their own concept of a "productivity enhancing redistribution" and demonstrate that such a redistribution is possible in their model. The above authors demonstrate in a general sense that equity and efficiency are not separable issues. However, they do not demonstrate the specific claim made by Stiglitz: that wealth distribution can affect whether or not an economy is or is not Pareto efficient through its effect on the existence of moral hazard.

Additionally, the above authors seem to suffer from a common mistake in the literature, highlighted by Ray and Singh (1998), by assuming that a model with a limited-liability constraint similar to Shetty (1988) can lead to sharecropping contracts. As Ray and Singh demonstrate limited-liability similar to that first proposed by Shetty (1988) does not lead to the existence of sharecropping. This error does not make limited-liability models any less useful for

the purpose of demonstrating the effect of wealth distribution on efficiency, but is an important distinction.

As described above Ray and Singh demonstrate that limited-liability in a basic form is not an explanation of sharecropping. However, their model can still be useful for the purpose of demonstrating the effect of wealth distribution on efficiency because moral hazard still arises in their model due to the limited-liability clause itself. Thus, I am able use their model to demonstrate more formally Stiglitz's assertion that whether or not an economy is Pareto efficient depends on the distribution of wealth. This result occurs because whether or not moral hazard occurs depends on the wealth level of the tenant.

However, Ray and Singh's model only includes one good, labor, as an input for production, which affects the ability to design Pareto improvements and thus affects the constrained Pareto efficiency of equilibrium. This result mirrors earlier results of Stiglitz and some of his coauthors. Specifically, in the Greenwald and Stiglitz (1986) result described above it is only possible to design a Pareto-improving tax-and-subsidy scheme because there are multiple goods interacting together. When there is only one good, no such Pareto improvements are possible, so while the economy is not first-best Pareto efficient, it is constrained Pareto efficient. Specifically, using the Ray and Singh model it is possible to demonstrate that some distributions are first-best efficient and some are not, but all such distributions will lead to equilibriums that are constrained Pareto efficient.

An example in Stiglitz (1994) was helpful in searching for a way to adapt the Ray and Singh model to allow for Pareto improving tax-and-subsidy schemes. Stiglitz proposes a situation where a sharecropper has two inputs, their labor and fertilizer which they purchase on the market. Further he proposes that additional fertilizer use increases the marginal product of

labor. In such a situation, "the landlord benefits from the tenant's use of fertilizer, not only directly, through the increased direct yield, but also indirectly if the fertilizer increases the marginal product of the tenant, and accordingly the tenant works harder" (p. 59). This is a surprising result that only occurs because of the existence of imperfect information that leads to moral hazard. Because moral hazard leads the tenant to undersupply labor, a subsidy on fertilizer that increases the labor of the tenant will bring the labor level closer to the efficient level, whereas if information was perfect the labor level would already be at the efficient level.

Given the Greenwald and Stiglitz result, introducing a second good of some kind seems like it would allow for the demonstration of the stronger result, that distribution affects whether or not an economy is constrained Pareto efficient. Combining this with the Stiglitz example of a fertilizer subsidy, I chose to modify the Ray and Singh model by adding fertilizer as a second input that increased the marginal product of labor. Given the literature cited above it seemed that in such a model I would be able to demonstrate that when moral hazard occurred a Pareto improving tax-and-subsidy scheme could be designed. Thus I would be able to demonstrate that certain wealth distributions that did not lead to moral hazard would be Pareto efficient, while distributions that led to moral hazard would not even be constrained Pareto efficient.

Overall, it appears that the literature is lacking a formal model that demonstrates the effect, described by Stiglitz, that distribution has on Pareto efficiency through its effect on the existence of relationships that bring about moral hazard. Thus in this paper I will use the Ray and Singh model of land tenancy with limited liability constraints to demonstrate this effect. First, I will use the unadapted model to demonstrate that wealth distribution affects whether or not an economy is first-best Pareto efficient. Second, by introducing fertilizer into the model, I am able to demonstrate the stronger conclusion. Using this model I demonstrate that under certain

conditions, whether or not an economy is constrained Pareto-efficient depends on the distribution of wealth. Further, I outline specifically what conditions must hold for this to be the case.

3. Unadapted Model

Ray and Singh (1998)[2] present a model of land tenancy with a limited liability constraint, meaning that for "for sufficiently adverse realizations of output the rent cannot be paid completely, the liability of the tenant is limited by his amount of wealth." In other words if particularly bad weather or other conditions lead to a bad harvest, the tenant may default on their payments due. As emphasized by Shetty (1988), who formulated an earlier version of the model, moral hazard problems similar to those found in sharecropping can arise in the presence of a limited liability constraint, when default is possible. Importantly, as will be demonstrated in the model, the existence of moral hazard and inefficient incentives depends on the wealth of the tenant. I will present their model, and then use it to analyze the Pareto efficiency of different distributions of wealth. As described above, I will demonstrate that in this model whether or not an economy is first-best Pareto efficient can depend on the distribution of wealth.

Set-up of the Model

In the model both tenants and landlord are assumed to be risk neutral. The production function representing a tenants production of crop is defined as $Q = \theta \cdot Q(L)$, where Q'>0,

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[2] Ray and Singh present two variations of the model. In the first variation the cropshare, the share of output that the tenant receives, is allowed to be greater than one. In this variation by pushing the cropshare above one contracts for less wealthy tenants can be pushed to efficiency by setting the cropshare greater than one. However, they present a number of reasons that it is reasonable to assume that in reality the cropshare would be constrained to be less than one. Thus, they present a second variation of the model with cropshare constrained, which results in the very different conclusions I describe. I will be using the second variation.

Q''<0, $L \geq 0$, Q(L)=0,[3] and θ is a random variable with distribution function $F, \in [\underline{\theta}, \overline{\theta}]$. θ represents the randomness in agricultural output due to weather and other circumstances. Let W be the initial wealth of the tenant. Landlords are able to observe the wealth level of tenants.

Tenants and landlords are assumed to bargain over the form of the contract. It is assumed further that landlords cannot determine the work effort of their tenants and that monitoring is too costly to be implemented. Therefore contracts are limited to be either rental contracts or share contracts, or some hybrid form in between. In the model it is assumed that tenants compete for landlords, but as Ray and Singh emphasis the results would be the same if the reverse were assumed. The tenant's income from the contract is defined as $\alpha \cdot \theta \cdot Q(L) - C$, where α is the tenant's crop-share, the proportion of output the tenant keeps as determined by the contract, and C is the fixed rental component of the contract (when C is positive it represent the payment made by the tenant to the landlord). Similarly, the tenants payment to the landlord is:

$(1-\alpha) \cdot \theta \cdot Q(L) + C$.

To represent the limited liability constraint θ_1 is defined as a value of θ such that the tenant cannot make the agreed upon payment, defined above as $(1-\alpha) \cdot \theta \cdot Q(L) + C$, for any value of θ less than or equal to θ_1. In other words, θ_1 represents the weather and other conditions that would lead to the tenant defaulting under those conditions or any worse conditions. Formally θ_1 is defined as follows:

$$\theta_1(\alpha, C, L; W): \begin{cases} \alpha\theta_1 Q(L) + W = C \\ \underline{\theta} \ if \ \alpha\underline{\theta}Q(L) + W \geq C \end{cases}$$

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[3] Ray and Singh do not include this explicitly as an assumption, but it is used (and necessary) later in their paper. It is also a very reasonable assumption: that if the tenant puts in no labor they will yield no output.

The value of θ_1 for a particular tenant depends on that tenants wealth level W.

Specifically, θ_1 is decreasing in W ceteris paribus, that is wealthier tenants require worse

conditions to force a default because they have more wealth to fall back on.

In cases of good weather conditions, where θ is greater than θ_1, the tenant pays the landlord the

agreed upon payment, $(1-\alpha) \cdot \theta \cdot Q(L) + C$. In the case of adverse conditions, where θ is less

than θ_1, the tenant by definition cannot make the agreed upon payment and instead pays all that

they are capable, their initial wealth plus all of their output, to the landlord. Additionally, labor is

assumed to be measured in disutility units, that is the disutility of labor L is just L, this merely

makes the algebra simpler. Therefore the expected earning of a tenant is:

$$\int_{\theta_1}^{\overline{\theta}} [\alpha\theta Q(L) - C] dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W) dF(\theta) - L$$

Similarly the expected earning of a landlord is:

$$\int_{\theta_1}^{\overline{\theta}} [(1-\alpha)\theta Q(L) + C] dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q(L) + W] dF(\theta)$$

For future reference and comparison it will be helpful to define L[**] as the first-best

efficient labor choice, that which would be chosen in the absence of asymmetric information and

that which maximizes the total expected earnings of the tenant and landlord. The total expected

earnings of tenant and landlord is just $Q(L) - L$. Therefore the first order condition for

maximization of total output is $Q'(L) - 1 = 0$. Therefore L[**] is defined by $Q'(L^{**}) = 1$.

The Contracting Problem

The contracting problem is for the landlord to maximize his or her earnings subject to a

number of constraints. The limited liability constraint is already built into the model and thus

does not enter the maximization problem in the usual way. However, there are 3 constraints to

the maximization problem. The first constraint is a participation constraint (PC) that represents the fact that the earnings of the tenant must be equal to or greater than some reservation income or opportunity cost, defined as K. The second constraint is an incentive constraint (IC) that represents the fact that the tenant chooses labor to maximize his or her expected earnings. This constraint is derived from the first order condition for a tenant's maximization problem. Finally, the last constraint is a share constraint (SC) that restricts $\alpha \leq 1$. One reason that the cropshare, α, could be expected to be constrained in reality is that when the cropshare is greater than one the landlord must acquire some extra crop to give to the tenant (Shetty 1988). This is a much more involved process than the tenant merely keeping an agreed upon portion of their output and requires the landlord to be trusted to deliver. Further, when the cropshare is greater than 1 a landlord may have an incentive to tamper with the contract by destroying output, whereas when cropshare is less than 1 both tenant and landlord gain from increases in output (Ray and Singh 2001).

Incorporating the above constraints the contracting problem can be represented as follows:

$$\underset{\{\alpha, C, L\}}{\text{Maximize}} : \quad \int_{\theta_1}^{\overline{\theta}} [(1-\alpha)\theta Q(L) + C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q(L) + W]dF(\theta)$$

Subject to: (PC) $\int_{\theta_1}^{\overline{\theta}} [\alpha\,\theta Q(L) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1}(-W)dF(\theta) - L \geq K$

(IC) $\int_{\theta_1}^{\overline{\theta}} [\alpha\,\theta Q'(L)]dF(\theta) - 1 = 0$

(SC) $\alpha \leq 1$

Following the procedure of Ray and Singh, the incentive constraint (IC) can be solved for L and then inserted into the objective function. We can then set-up a Lagrangian using λ and μ as Lagrange multipliers for (PC) and (SC) respectively:

$$F(\lambda, \mu, \alpha, C, L) = \int_{\theta_1}^{\overline{\theta}}[(1-\alpha)\theta Q(L) + C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} \theta Q(L) + W]dF(\theta) + \lambda \cdot$$

$$\left[\int_{\theta_1}^{\overline{\theta}}[\alpha\,\theta Q(L) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} \theta Q(L) + W]dF(\theta) - K\right] + \mu \cdot (1-\alpha)$$

The first order conditions for this Lagrangian are then as follows:

$$F_\alpha(\lambda, \mu, \alpha, C, L) = \int_{\theta_1}^{\overline{\theta}}[-\theta Q(L) + (1-\alpha)\theta Q^{'}(L)L_\alpha]dF(\theta) + \int_{\underline{\theta}}^{\theta_1}[\theta Q^{'}(L)L_\alpha]dF(\theta) +$$

$$\lambda\left[\int_{\theta_1}^{\overline{\theta}}[\theta Q(L) + \alpha Q'(L)L_\alpha]dF(\theta) - L_\alpha\right] - \mu = 0$$

and

$$F_C(\lambda, \mu, \alpha, C, L) = \int_{\theta_1}^{\overline{\theta}} dF(\theta) - \lambda \int_{\theta_1}^{\overline{\theta}} dF(\theta) = 0$$

At this point Ray and Singh consider three "exhaustive and mutually exclusive cases" depending on which constraints bind. I will follow this method.

Case 1: Participation constraint does not bind

In this case we assume that the participation constraint (PC) does not bind. By definition because (PC) does not bind $\lambda=0$. The first order condition for C then becomes:

$$F_C(\lambda, \mu, \alpha, C, L) = \int_{\theta_1}^{\overline{\theta}} dF(\theta) = 0$$

From this we can conclude that $\theta_1 = \overline{\theta}$. In other words, the tenant will default under all conditions. Given this fact, we return to the tenant's maximization of his or her expected earnings (by choosing L):

$$\begin{array}{c} Max \\ (L) \end{array} \qquad \int_{\theta_1}^{\overline{\theta}}[\alpha\theta Q(L) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1}(-W)dF(\theta) - L$$

Given that $\theta_1 = \overline{\theta}$, the above maximization becomes:

$$\begin{array}{c} Max \\ (L) \end{array} \qquad -W - L$$

Given that L is constrained such that $L \geq 0$, the tenants maximizing labor choice will be: L^*=0. That is, to maximize his or her expected earning the tenant chooses to not put out any effort, because his or her wealth level is so low that they will default under any circumstances. Given that Q(L*)=0, the original definition of θ_1, $\alpha\theta_1 Q(L^*) + W = C$, becomes W=C. That is, the fixed rent is equal to the tenants wealth level.

Now given that the (PC) does not bind, $\theta_1 = \overline{\theta}$, and L^*=0, (PC) becomes:

$$\int_{\underline{\theta}}^{\overline{\theta}} [\alpha \theta Q(L) - C]dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} (-W)dF(\theta) > K$$

Which simplifies to, $-W > K$, or $W < -K$. Therefore in this case, the tenants wealth level is less than (-K), in other words the tenant is indebted.

Finally, since $\theta_1 = \overline{\theta}$ the landlords expected earnings becomes:

$$\int_{\underline{\theta}}^{\overline{\theta}} [(1-\alpha)\theta Q(L) + C]dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} [\theta Q(L) + W]dF(\theta)$$

Given that $L^* = 0$, and thus Q(L)=0, this simplifies to just W. From above we know that $W < -K$, so the landlords expected earnings would be less than (-K). If the landlord did not lease the plot at all and just left it fallow his expected earnings would be 0, which is greater than (-K). Therefore it is the maximizing landlord's interest not to rent the land at all. This result arises because the landlord is so poor that they expect to default under any conditions and thus have no incentive to put out any effort. In conclusion then, Case 1 is not a solution to the contracting problem, no contract will arise and the landlord will leave the plot fallow.

Case 2: Participation constraint binds and $\alpha < 1$

For this case we assume that the participation constraint (PC) binds as an equality and that the cropshare constraint (CC) does not bind at equality, that is $\alpha < 1$. By definition then, $\mu = 0$.

Given that the participation constraint (PC) binds we have:

$$\int_{\theta_1}^{\overline{\theta}} [\alpha\,\theta Q(L) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W)dF(\theta) - L = K$$

It is clear that if $\theta_1 = \overline{\theta}$, (PC) would become $-W - L = K$. Because we have assumed that $K > 0$, we would either have that the tenant is indebted or the above expression would be a contradiction. Either way we can conclude that for this case to be a solution to the contracting problem $\theta_1 < \overline{\theta}$.

Now we can examine the first order condition for C:

$$F_C(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\overline{\theta}} dF(\theta) - \lambda \int_{\theta_1}^{\overline{\theta}} dF(\theta) = 0$$

Which we can rearrange to:

$$\int_{\theta_1}^{\overline{\theta}} dF(\theta)\,(1 - \lambda) = 0$$

Given that $\theta_1 < \overline{\theta}$, we can conclude that $\int_{\theta_1}^{\overline{\theta}} dF(\theta) > 0$, and thus that $\lambda = 1$. Therefore the first-order condition for α becomes:

$F_\alpha(\lambda, \mu, \alpha, C, L)$

$$= \int_{\theta_1}^{\overline{\theta}} [-\theta Q(L) + (1-\alpha)\theta Q'(L)L_\alpha]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q'(L)L_\alpha]dF(\theta)$$

$$+ \int_{\theta_1}^{\overline{\theta}} [\theta Q(L) + \alpha Q'(L)L_\alpha]dF(\theta) - L_\alpha = 0$$

After some rearranging this becomes:

$$Q'(L)L_\alpha \left(1 - \alpha \int_{\theta_1}^{\overline{\theta}} \theta dF(\theta) \right) + L_\alpha \left(\int_{\theta_1}^{\overline{\theta}} \alpha \theta Q'(L)dF(\theta) - 1 \right) = 0$$

Using the incentive constraint (IC) we see that the second term is equal to zero and drops out.

We are thus left with:

$$Q'(L)L_\alpha \left(1 - \alpha \int_{\theta_1}^{\overline{\theta}} \theta \, dF(\theta) \right)$$

which, given that $Q'(L) > 0$ and $L_\alpha > 0$, implies:

$$\alpha = \frac{1}{\int_{\theta_1}^{\overline{\theta}} \theta dF(\theta)}$$

We know from the definition of the distribution of θ, that $\int_{\theta_1}^{\overline{\theta}} \theta dF(\theta) \leq 1$. Therefore the above

equation tells us that $\alpha \geq 1$, which is a contradiction given that the case under consideration

assumes that $\alpha < 1$. Therefore this case does not arise as a solution to the contracting problem.

Together the fact that Case 1 and 2 lead to contradictions, demonstrates that situations where the

share constraint does not bind cannot be solutions to the contracting problem. A true share

contract (where the tenants share $\alpha < 1$) will not arise in this model. If a solution does arise for

the final case it will be a fixed rent contract. This is one of the main conclusion of Ray and Singh:

that a limited liability constraint such as that used in their model cannot be an explanation for

sharecropping.

Case 3: Participation constraint binds and $\alpha = 1$

In the third and final case we will assume that the participation constraint (PC) and

cropshare constraint (CC) both bind at equality. That is, $\alpha = 1, \lambda \geq 0,$ and $\mu \geq 0$. As in Case 2,

it is evident from the first order condition for C that $\lambda = 1$. Therefore the first order condition for α becomes:

$$F_\alpha(\lambda, \mu, \alpha, C, L) = \int_{\theta_1}^{\overline{\theta}} [-\theta Q(L) + (1-\alpha)\theta Q'(L)L_\alpha]dF(\theta) + \int_{\underline{\theta}}^{\theta_1}[\theta Q'(L)L_\alpha]dF(\theta) +$$

$$\int_{\theta_1}^{\overline{\theta}}[\theta Q(L) + \alpha Q'(L)L_\alpha]dF(\theta) - L_\alpha - \mu = 0$$

With some rearranging and cancellation this becomes:

$$L_\alpha\left(\int_{\theta_1}^{\overline{\theta}}\theta\, Q'(L)dF(\theta) - 1\right) + \int_{\underline{\theta}}^{\theta_1}\theta Q'(L)L_\alpha dF(\theta) = \mu$$

Given that $\alpha = 1$, the incentive constraint (IC) becomes: $\int_{\theta_1}^{\overline{\theta}}\theta Q'(L)dF(\theta) - 1 = 0$. Thus the first term is equal to zero and drops out, leaving:

$$Q'(L)L_\alpha\int_{\underline{\theta}}^{\theta_1}\theta dF(\theta) = \mu$$

Which implies that $\mu \geq 0$. This is not a contradiction with the assumptions of the case. This case holds as a solution to the contracting problem. Thus we have shown that the only solution to the contracting problem is a case where $\alpha = 1$. In other words, in all contracts the tenant will get to keep all of their output, no sharecropping will arise in this model. While we have not yet shown the specific value of the fixed rent, C, it is clear that if the tenant is receiving all of the output and the landlord is receiving none, there must be some fixed rent paid from the tenant to the landlord. Following the strategy of Ray and Singh we will now examine two subcases of Case 3 that will give us greater insight into effect that wealth level has on contract form and efficiency.

Case 3(a): Participation constraint binds, $\alpha = 1$, and limited liability constraint does not bind

In this sub-case we will assume as in Case 3 that the participation constraint (PC) and the cropshare constraint (CC) both bind and equality. Further we will assume that the limited

liability constraint does not bind, that is $\theta_1 = \underline{\theta}$. Given that $\theta_1 = \underline{\theta}$ and $\alpha = 1$, the incentive

constraint (IC) becomes:

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta Q'(L) dF(\theta) - 1 = 0$$

Which simplifies to $Q'(L) = 1$, which means in this case L is the efficiency maximizing labor

choice L^{**}.

From the participation constraint (PC) we can determine the value of the fixed rent, C.

Given that $\theta_1 = \underline{\theta}$, $L=L^{**}$, and the participation constraint binds at equality, PC becomes:

$$\int_{\underline{\theta}}^{\bar{\theta}} [\theta Q(L^{**}) - C] \, dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} (-W) dF(\theta) - L^{**} = K$$

Which simplifies to:

$$C = Q(L^{**}) - L^{**} - K$$

The fixed rent paid to the landlord will be equal to the expected output minus the

disutility of labor and the tenants reservation outcome. Because the limited liability constraint is

assumed not to bind, the tenant will never default so the landlord will receive C in all conditions.

Because the participation constraint (PC) binds the expected earnings of the tenant is equal to

their reservation income, K.

To determine under what wealth levels, W, this case will occur we will examine the

definition of θ_1. Given that $\alpha = 1$ and the limited liability constraint binds, that is $\theta_1 = \underline{\theta}$, we

can obtain the following from the definition of θ_1:

$$\underline{\theta} Q(L) + W \geq C$$

Which becomes:

$$W \geq C - \underline{\theta} Q(L)$$

Which substituting in the above expression for C becomes:

$$W \geq Q(L^{**}) - L^{**} - K - \underline{\theta}Q(L^{**})$$

We can thus define a cutoff level of wealth, $\overline{W}$, such that this case (Case 3a) will arise for

tenants with $W \geq \overline{W}$. Because of the earlier results of this case, this is also the cutoff level such

that tenants with $W \geq \overline{W}$ will have contracts that lead them to put out the efficient labor level,

L^{**}, and thus will lead to the maximization of total output, in other words first-best efficiency.

This cutoff level $\overline{W}$ is defined as follows:

$$\overline{W} \equiv Q(L^{**}) - L^{**} - K - \underline{\theta}Q(L^{**})$$

We will now proceed to the second sub-case.

Case 3(b): Participation Constraint Binds, $\alpha = 1$, and Limited Liability Constraint Binds

In this sub-case we will assume as in Case 3 that the participation constraint (PC) and the

cropshare constraint (CC) both bind and equality. Further, in contrast to Case 3(a) we will

assume that the limited liability constraint does bind. In other words, the tenant's wealth is low

enough that there is a chance of default under some conditions, formally, that is $\theta_1 > \underline{\theta}$.

Given that $\alpha = 1$ the incentive constraint (IC) becomes:

$$Q'(L) \int_{\theta_1}^{\overline{\theta}} \theta \, dF(\theta) - 1 = 0$$

Which can be rearraned to give us:

$$Q'(L) = \frac{1}{\int_{\theta_1}^{\overline{\theta}} \theta \, dF(\theta)}$$

Given the distribution of θ and the fact that $\theta_1 > \underline{\theta}$, we can conclude that $\int_{\theta_1}^{\overline{\theta}} \theta \, dF(\theta) <$

1. Thus the above expressions tells us that $Q'(L) > 1$. The tenant will not choose the efficient

labor level, L^{**}. Given that $Q''(L) < 0$, it is clear that the tenant in this case will choose a labor

level below the efficient level, that is $L^* < L^{**}$. The intuition behind this result is simple: in the case of default the tenant does not receive any of the output and thus does not receive any benefit from their labor input, therefore if a tenant has a low enough wealth level that there is some chance of default they will not have incentives to choose a labor level that maximizes total output. Thus in this case total output is not maximized due to moral hazard, the equilibrium contract for less wealthy tenants is not first best efficient.

Welfare Analysis

Given the above analysis of contract forms under different wealth levels, it is possible to execute a welfare analysis comparing the Pareto efficiency of different wealth distributions. I will demonstrate that under Ray and Singh's model the distribution of initial wealth affects the first-best Pareto efficiency of equilibrium, but not the constrained efficiency. As described earlier the model must be adapted to demonstrate the stronger result with constrained efficiency.

We will compare the welfare properties of two distributions, one where all tenants hold equal wealth and one where wealth is distributed unequally. For both distributions, the total number of tenants is equal to n and the total wealth of tenants is equal to $n\overline{W}$, the number of tenants times the cutoff wealth level. Under *Distribution A*, all tenants hold equal wealth, $\overline{W}$. Under *Distribution B* tenants are split into two equal sized groups of $\frac{n}{2}$ tenants. The first group has wealth, $W = \overline{W} + y$, where y is an arbitrary constant. The second group holds wealth, $W = \overline{W} - y$.

First, consider the welfare properties of *Distribution A*. Because all tenants hold wealth at least equal to $\overline{W}$, at equilibrium all contracts arising from this distribution will be of the form demonstrated in Case 3(a). Therefore, all tenants will choose the efficient labor level, L^{**}, that

maximizes total output for themselves and the landlord. All contracts will be first best efficient and therefore the equilibrium will be first-best Pareto efficient.

Next, we can consider the welfare properties of *Distribution B*. The more wealthy tenants, with $W = \overline{W} + y$, will receive contracts of the form demonstrated in Case 3(a), and will thus put out the efficient labor level, L^{**}, that maximizes total output for themselves and the landlord. However, the less wealthy tenants, with $W = \overline{W} - y$, will receive contracts of the form demonstrated in Case 3(b). Therefore, they will choose a level of labor less than that the efficient level, that is $L^* < L^{**}$, the total welfare of tenant and landlord will not be maximized at a first-best level. These contracts will then not be first-best efficient. We can then say that the equilibrium of the economy overall is not first best efficient.

Thus, we have the result that the initial distribution of wealth affects whether or not the equilibrium is *first-best* Pareto efficient. For one distribution, the equilibrium is first best efficient because no moral hazard occurs, and for another distribution the equilibrium is not first-best efficient because of the existence of moral hazard. However, both distributions are constrained Pareto efficient, taking the imperfect information as given no Pareto improvements are possible. In the following section I attempt to adapt the Ray and Singh model slightly to obtain the stronger result, that the *constrained* Pareto efficiency of equilibrium depends on the initial distribution of wealth.

4. Model with Addition of Fertilizer

I will now present a version of the Ray and Singh model that is adapted by the addition of a second input in the production function. This additional input can be understood as fertilizer. In this model, a tenant chooses not just an amount of labor but also can purchase fertilizer on the

market and use it on the fields. Additional fertilizer is assumed to increase output and also to increase the marginal productivity of labor. If a tenant uses more fertilizer it not only directly increases output, but also makes their labor more productive. This assumption is important because it will eventually allow for a subsidy on fertilizer that will not only lead a tenant to use more fertilizer but will also indirectly lead to the tenant choosing to put out more labor.

Just as labor was assumed to not be observable to the landlord, the amount of fertilizer that a tenant uses is also unobservable to the landlord. While the landlord could more easily observe or mandate how much fertilizer is purchased, it would be difficult for the landlord to ensure that the tenant actually used the fertilizer and did not sell it back on the market. In most other respects the model is the same as Ray and Singh, as shown below.

Set-up of the Model

As in the earlier model tenants and landlords are assumed to be risk neutral. A tenant's output is defined as $\theta Q(L, X)$, where L is labor, X is fertilizer and θ is a random variable representing randomness in output due to weather and other factors. It is assumed that $Q_L > 0$, $Q_{LL} < 0$, $Q_X > 0$, $Q_{XX} < 0$, and $Q_{LX} > 0$. θ has distribution function F, $\in \left[\underline{\theta}, \overline{\theta}\right]$, and E($\theta$)=1. Again, a tenant's wealth level is defined by W, and landlords are able to observe a tenants wealth level. The tenant's income from the contract is defined as $\alpha\theta Q(L, X) - C$. Similarly, the tenant's payment to the landlord is defined as $(1 - \alpha)\theta Q(L, X) + C$. The tenant is assumed to have a reservation income K. It is assumed that $K \geq 0$.

As before θ_1 is defined as the value of θ such that the tenant cannot make the agreed upon payment and will default when $\theta \leq \theta_1$. Formally θ_1 is defined as follows:

$$\theta_1(\alpha, C, L, X; W): \begin{cases} \alpha\theta_1 Q(L, X) + W = C \\ \underline{\theta} \text{ if } \alpha\underline{\theta}Q(L, X) + W \geq C \end{cases}$$

Tenants are assumed to purchase fertilizer at a constant market price, p, where $p > 0$. For simplicity, it is assumed that a tenant always has enough wealth to purchase the amount of fertilizer they choose to. However, it is assumed that the purchase of fertilizer takes place at the beginning of the period and thus is subtracted from the wealth level left by the end of the period. Therefore in the case of default the tenant pays to the landlord all their output plus their remaining wealth subtracting the amount they spent on fertilizer. As will be examined further this leads to an odd result where in the case of default the tenant experiences neither the benefit nor the cost of fertilizer use. The effect of this "double moral hazard" on the tenant choice of fertilizer is examined in more detail below.

Formally, the expected earning of the tenant is defined as follows:

$$\int_{\theta_1}^{\overline{\theta}} [\alpha\theta Q(L,X) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W + pX)dF(\theta) - L - pX$$

Similarly the expected earning of the landlord is:

$$\int_{\theta_1}^{\overline{\theta}} [(1-\alpha)\theta Q(L,X) + C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q(L,X) + W - pX]dF(\theta)$$

Again, we will define L^{**}, and now X^{**} additionally, as the first-best efficient labor and fertilizer choices that maximize total expected earnings. Total expected earnings is:

$$Q(L,X) - L - pX$$

Thus the maximization of expected earnings yields the following first order conditions for maximization:

FOC (L): $\quad\quad\quad\quad Q_L - 1 = 0$

FOC (X): $\quad\quad\quad\quad Q_X - p = 0$

Therefore we can define L^{**} and X^{**} as those values that satisfy: $Q_L(L^{**}, X^{**}) = 1$ and $Q_x(L^{**}, X^{**}) = p$. Importantly, these two values come as a pair, that is for X and L to be at the

efficient level the pair of values of L^{**} and X^{**} must together satisfy both of the above conditions.

For example if $Q_L(L, X) = 1$, but $Q_x(L, X) \neq p$ neither fertilizer or labor are at the efficient level

L^{**} or X^{**}.

The Contracting Problem

As before the landlord maximizes his or her earnings subject to a number of constraints. The participation constraint (PC) and share constraint (SC) are the same concept as above. With the addition of fertilizer to the model there are now two incentive constraints, one for labor (ICL) and one for fertilizer (ICX). These constraints are derived from the tenant's maximization of his earnings, as shown below:

$$\underset{(L, X)}{Max} \qquad \int_{\theta_1}^{\overline{\theta}} [\alpha\theta Q(L, X) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W + pX)dF(\theta) - L - pX$$

The first order conditions for maximization are thus:

FOC(L): $\qquad \int_{\theta_1}^{\overline{\theta}} \alpha\theta Q_L(L, X)dF(\theta) - 1 = 0$

FOC(X): $\qquad \int_{\theta_1}^{\overline{\theta}} [\alpha\theta Q_X(L, X) - p]dF(\theta) = 0$

These become the two incentive constraints ICL and ICX respectively. Thus the contracting problem can be represented by the landlord's maximization subject to constraints as follows:

$$\underset{(\alpha, C)}{Max} \qquad \int_{\theta_1}^{\overline{\theta}} [(1 - \alpha)\theta Q(L, X) + C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q(L, X) + W - pX]dF(\theta)$$

Subject to:

(PC) $\qquad \int_{\theta_1}^{\overline{\theta}} [\alpha\theta Q(L, X) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W + pX)dF(\theta) - L - pX \geq K$

(ICL) $\qquad \int_{\theta_1}^{\overline{\theta}} \alpha\theta Q_L(L, X)dF(\theta) - 1 = 0$

(ICX) $\int_{\theta_1}^{\overline{\theta}}[\alpha\theta Q_X(L,X) - p]dF(\theta) = 0$

(CC) $\alpha \leq 1$

Following Ray and Singh the incentive constraints (ICL) and (ICX) are solved for L and

X respectively and substituted into the objective function. The remaining constraints can be

incorporated into the maximization through a Lagrangian. The Lagrangian, with λ and μ as

Lagrange multiplier for (PC) and (SC) respectively, is below:

$$F(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\overline{\theta}}[(1-\alpha)\theta Q(L,X) + C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1}[\theta Q(L,X) + W - pX]dF(\theta)$$

$$+ \lambda\left(\int_{\theta_1}^{\overline{\theta}}[\alpha\theta Q(L,X) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1}(-W + pX)dF(\theta) - L - pX - k\right) + \mu(1$$

$$- \alpha)$$

The first-order conditions for the Lagrangian, in terms of α and C are thus:

$$F_\alpha(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\overline{\theta}}[-\theta Q(L,X) + (1-\alpha)\theta(Q_L L_\alpha + Q_X X_\alpha)]$$

$$+ \int_{\underline{\theta}}^{\theta_1}[\theta(Q_L L_\alpha + Q_X X_\alpha) - pX_\alpha]dF(\theta)$$

$$+ \lambda\left(\int_{\theta_1}^{\overline{\theta}}[\theta Q(L,X) + \alpha\theta(Q_L L_\alpha + Q_X X_\alpha)]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta) - L_\alpha - pX_\alpha\right)$$

$$- \mu = 0$$

$$F_C(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\overline{\theta}}dF(\theta) - \lambda\int_{\theta_1}^{\overline{\theta}}dF(\theta) = 0$$

As above, now that the basic contracting problem is established we can examine three

cases, with different assumptions about which constraints bind. As before Case 1, in which the

participation constraint (PC) is assumed not to bind, does not hold as a solution to the contracting

problem. The reasoning for this result is the same as for the unadapted model, and is demonstrated formally in an appendix.

However, the addition of fertilizer, and specifically the different way that limited-liability affects fertilizer, leads to the result that Case 2 and Case 3 can both hold under different conditions. In Case 2 the participation constraint is assumed to bind but the share constraint is assumed not to bind, that is $\alpha < 1$. In Case 3 the participation constraint is again assumed to bind and the share constraint is also assumed to bind, that is $\alpha = 1$. Thus the fact that Case 2 and Case 3 can hold, demonstrates that sharecropping will arise under certain conditions and not arise under others. This is in contrast to the result of Ray and Singh outlined above for a model without fertilizer, that only Case 3 can hold and thus no sharecropping will arise.

This is an interesting result and brings back on the table the possibility of limited-liability as an explanation for sharecropping. However, to demonstrate the effect that is the focus of this paper, that the constrained Pareto-efficiency of equilibrium can depend on the distribution of wealth, I will proceed by assuming that Case 3 holds, and thus $\alpha = 1$ and no sharecropping arises. A brief examination of the conditions that cause each case to hold is left to a later section on caveats and areas for further research.

Case 3: participation constraint binds, cropshare constraint binds

We assume that Case 3 binds, thus the participation constraint (PC) and the cropshare constraint (CC) bind. Now we proceed to examine the different results of this case depending on whether or not the limited-liability constraint binds.

Case 3(a): Participation constraint binds, share constraint does not bind, limited liability constraint does not bind

In this sub-case we make all the same assumptions as Case 3, plus the additional assumption that the limited liability constraint does not bind. In other words, we assume that the tenant's wealth is high enough that they have no risk of default, even under the worst possible weather conditions he or she will still be able to pay the agreed upon rent. Formally, this assumption is represented by $\theta_1 = \underline{\theta}$. As will be demonstrated below, when the tenant has no chance of default no moral hazard will occur, and thus the contract will result in first-best efficiency.

Given that $\theta_1 = \underline{\theta}$ and $\alpha = 1$, the incentive constraint for labor (ICL) becomes:

$$\int_{\underline{\theta}}^{\overline{\theta}} \theta Q_L(L,X)dF(\theta) - 1 = 0$$

Which simplifying and solving for Q_L yields :

$$Q_L(L,X) = 1$$

Similarly, given that $\theta_1 = \underline{\theta}$ and $\alpha = 1$, the incentive constraint for fertilizer (ICX) becomes:

$$\int_{\underline{\theta}}^{\overline{\theta}} [\theta Q_X(L,X) - p]dF(\theta) = 0$$

Which simplifying and solving for Q_X yields:

$$Q_X(L,X) = p$$

Together $Q_L(L,X) = 1$ and $Q_X(L,X) = p$ satisfy the conditions for efficient labor and fertilizer choice. Thus, $L^* = L^{**}$ and $X^* = X^{**}$. The equilibrium labor and fertilizer choices will both be first-best efficient, and therefore the contract as a whole is first-best efficient and total output is maximized. This result is intuitive. In this case we have assumed that the tenant is wealthy enough to have no risk of default. Therefore there is no problem of moral hazard and the equilibrium will be first-best Pareto efficient.

As before, we can define formally a cutoff wealth level above which this case will occur. In other words, we can define how wealthy a tenant must be to have no chance of default and therefore have a contract that leads to first-best efficiency. This is a multi-step process: I will begin by using the participation constraint (PC) to define a value of C, then I will use the definition of θ_1 to obtain an expression of W, and finally I will substitute the expression for C into the expression of W.

First examine the participation constraint (PC) given that $\theta_1 = \underline{\theta}$:

$$\int_{\underline{\theta}}^{\overline{\theta}} [\alpha\theta Q(L^{**},X^{**}) - C]dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} (-W + pX^{**})dF(\theta) - L^{**} - pX^{**} \geq K$$

Which simplifies to:

$$Q(L^{**},X^{**}) - C - L^{**} - pX^{**} = K$$

Which solving for W yields:

$$C = Q(L^{**},X^{**}) - L^{**} - pX^{**} - K$$

Now we will examine the definition of θ_1, in order to obtain an expression for W. Given that $\alpha = 1$, $L^* = L^{**}$, and $X^* = X^{**}$, the definition of θ_1 becomes:

$$\underline{\theta}Q(L^{**},X^{**}) + W \geq C$$

Solving for C this becomes:

$$W \geq -\underline{\theta}Q(L^{**},X^{**}) + C$$

Substituting the expression for C obtained above this becomes:

$$W \geq Q(L^{**},X^{**}) - L^{**} - pX^{**} - K - \underline{\theta}Q(L^{**},X^{**})$$

We can define a cutoff level $\overline{W}$, such that this case only occurs if $W \geq \overline{W}$. Given the above expression we define $\overline{W}$ as:

$$\overline{W} \equiv Q(L^{**},X^{**}) - L^{**} - pX^{**} - K - \underline{\theta}Q(L^{**},X^{**})$$

Tenants with $W \geq \overline{W}$ will have no chance of default, and thus no moral hazard will arise in their contracts. Therefore for these tenants and their landlords (as in the unadapted model) the equilibrium will be first-best Pareto efficient, it will maximize total output.

For the welfare analysis below it will be helpful to have an expression of the landlord's expected earnings for this case. Given that $\theta_1 = \underline{\theta}$ and $\alpha = 1$, the expression for the landlord's expected earning's becomes:

$$\int_{\underline{\theta}}^{\overline{\theta}} C dF(\theta) + \int_{\underline{\theta}}^{\theta} [\theta Q(L,X) + W - pX]dF(\theta)$$

Which simplifies to just C. Thus the tenants expected earnings is equal to the expression above for C, which is:

$$Q(L^{**}, X^{**}) - L^{**} - pX^{**} - K$$

Case 3(b): Participation constraint binds, share constraint does not bind, limited liability constraint does bind

For this case we will assume as in Case 3 that the participation constraint (PC) and the share constraint (SC) both bind at equality. Additionally, we will assume that the limited liability constraint does not bind, that is the tenant's wealth is low enough that he or she has a chance of defaulting under some adverse conditions. Formally, that is $\theta_1 > \underline{\theta}$. As I will show, in this case, as in the unadapted model, equilibrium will not be efficient because having some probability of default leads to moral hazard and thus an inefficient choice of labor and fertilizer.

First we examine the incentive constraint for labor (ICL), given that $\alpha = 1$:

$$\int_{\theta_1}^{\overline{\theta}} \theta Q_L(L,X)dF(\theta) - 1 = 0$$

This can be solved for Q_L, yielding the following expression:

$$Q_L(L,X) = \frac{1}{\int_{\theta_1}^{\overline{\theta}} \theta dF(\theta)}$$

Given that $\int_{\theta_1}^{\overline{\theta}} \theta dF(\theta) < 1$, this implies that $Q_L(L,X) > 1$. Thus $L^* < L^{**}$. As in the

unadapted model, in this case the tenant will undersupply labor. Under certain conditions the

tenant will default and not receive any benefit of labor, but will still have to bear the cost of labor

disutility. Thus the choice of labor that maximizes the tenants output is lower than the choice of

labor that would maximize total output.

Now we can examine the incentive constraint for fertilizer (ICX), given that $\alpha = 1$:

$$\int_{\theta_1}^{\overline{\theta}} [\theta Q_X(L,X) - p]dF(\theta) = 0$$

Solving this for Q_X yields:

$$Q_X(L,X) = p \cdot \frac{\int_{\theta_1}^{\overline{\theta}} dF(\theta)}{\int_{\theta_1}^{\overline{\theta}} \theta dF(\theta)}$$

It can be demonstrated that $\int_{\theta_1}^{\overline{\theta}} \theta dF(\theta) > \int_{\theta_1}^{\overline{\theta}} dF(\theta)$ when $\theta_1 > \underline{\theta}$. Thus $\frac{\int_{\theta_1}^{\overline{\theta}} dF(\theta)}{\int_{\theta_1}^{\overline{\theta}} \theta dF(\theta)} < 1$

and the above expression implies that $Q_X < p$. This implies that the tenant will oversupply

fertilizer, that $X^* > X^{**}$. This is an interesting result that arises because of the "double moral

hazard" effect on fertilizer, where in the case of default the tenant experiences neither the benefit

nor the loss from additional fertilizer use.

The intuition behind this result is somewhat complex. At first blush it may seem

reasonable to assume that the two moral hazard-like effects would cancel out, and fertilizer

choice would be efficient, but this does not turn out to be the case. The result that fertilizer is

oversupplied depends on the fact that conditions that lead to default, where the tenant does not

care what level of fertilizer they supply, occur for low values of θ. On the other hand, when the tenant's choice of fertilizer does affect them, the values of θ will be higher. Thus, the marginal product of fertilizer is higher for the cases where the tenant will not default, than the average marginal product of fertilizer for all cases. Since, the tenant chooses their fertilizer uses based on these cases they will oversupply fertilizer, compared to the efficient level of fertilizer that would maximize total output.

For the welfare analysis below it will be helpful to have an expression for the landlord's expected earnings. This is a two step process: first we will use the participation constraint (PC) to obtain an expression for $\int_{\theta_1}^{\overline{\theta}} C dF(\theta)$. Then we will substitute this expression into the expression for the landlord's expected earnings.

First, we will reexamine the participation constraint (PC) given that $\alpha = 1$:

$$\int_{\theta_1}^{\overline{\theta}} [\theta Q(L,X) - C] dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W + pX) dF(\theta) - L - pX - K = 0$$

Rearranging this expression yields:

$$\int_{\theta_1}^{\overline{\theta}} C dF(\theta) = \int_{\theta_1}^{\overline{\theta}} \theta Q(L,X) dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W + pX) dF(\theta) - L - pX - K$$

Now, given that $\alpha = 1$ the expression for the landlord's expected earnings becomes:

$$\int_{\theta_1}^{\overline{\theta}} C dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q(L,X) + W - pX] dF(\theta)$$

Substituting the above expression for $\int_{\theta_1}^{\overline{\theta}} C dF(\theta)$ yields:

$$\int_{\theta_1}^{\overline{\theta}} \theta Q(L,X) dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W + pX) dF(\theta) - L - pX - K + \int_{\underline{\theta}}^{\theta_1} [\theta Q(L,X) + W - pX] dF(\theta)$$

Which simplifies to just:

$$Q(L, X) - L - pX - K$$

This is the same expression as above in Case 3(a), except the values for L and K are not the optimal levels L** and K** as above.

Welfare Analysis

As before, for the unadapted Ray and Singh model, now that we have analyzed contract forms under different wealth levels, it is possible to execute a welfare analysis comparing the Pareto efficiency of different wealth distributions. I will demonstrate that under the adapted model the distribution of initial wealth can be shown under certain conditions to affect not only the first-best Pareto efficiency of equilibrium but also the constrained Pareto efficiency.

Again, we will compare the welfare properties of two distributions, one where all tenants hold equal wealth and one where wealth is held unequally. The distributions to be compared are the same as above. There are assumed to be *n* tenants in both distributions. Under *Distribution A*, all tenants hold equal wealth, $\overline{W}$. Under *Distribution B* tenants are split into two equal sized groups of $\frac{n}{2}$ tenants. The first group has wealth, $W = \overline{W} + y$, where y is an arbitrary constant. The second group holds wealth, $W = \overline{W} - y$.

First, consider the welfare properties of *Distribution A*. Because all tenants hold wealth at least equal to $\overline{W}$, at equilibrium all contracts arising from this distribution will be of the form demonstrated in Case 3(a). Therefore, all tenants will put out efficient labor level, L**, and efficient fertilizer X**. Total output will thus be maximized. All contracts will be first best efficient and therefore the equilibrium will be first-best Pareto efficient. This result is the same as in the unadapted model.

Next, we can consider the welfare properties of *Distribution B*. The more wealthy tenants, with $W = \overline{W} + y$, will receive contracts of the form demonstrated in Case 3(a), and will thus put out the efficient labor level, L^{**}, that maximizes total output for themselves and the landlord. However, the less wealthy tenants, with $W = \overline{W} - y$, will receive contracts of the form demonstrated in Case 3(b). Therefore, as in the unadapted model, they will put out labor level less than that the efficient labor level, that is $L^* < L^{**}$. Further they will choose a fertilizer level that is greater than the efficient level, that is $X^* > X^{**}$. Thus the total welfare of tenant and landlord will not be maximized at a first-best level. These contracts will then not be first-best efficient and the economy overall is not first-best efficient.

So far this result is the same as in the unadapted model without fertilizer, however, with the addition of fertilizer we can additionally demonstrate the stronger result that *Distribution B* may not even be constrained Pareto efficient. In other words, even taking the imperfect information as given there may be a possible Pareto improvement. Specifically, we can demonstrate that under certain conditions a subsidy on fertilizer, paid for by a tax on landlord's earnings will lead to a Pareto improvement.

Tax-and-subsidy Scheme

The Pareto improving tax and subsidy scheme consist of a tax on landlords earnings and a subsidy on the price of fertilizer. It will be demonstrated that such a tax-and-subsidy scheme makes landowners with tenants below the cutoff wealth level better off, and leads to no change in earnings for tenants of all wealth levels, and for landlords with tenants above the cutoff wealth level. Therefore it is a Pareto-improvement because it makes one group better off without making anyone else worse off.

We assume that the government has the same information problems as the landlord, that is they cannot observe the amount of labor and fertilizer the tenant uses. However, as emphasized by Greenwald and Stiglitz (1987) it is reasonable to assume that there are certain things the government can do that individuals cannot. Specifically, in this model we assume that the government is able to enforce a market-wide subsidy on fertilizer. This is important because, a smaller-scale attempt by the landlord to subsidize fertilizer just for their landlord would be fraught by problems stemming from the fact that the landlord cannot observe the tenants actual fertilizer use. Specifically, if a small-scale subsidy by a single landlord was instituted the tenant would have an incentive to purchase additional fertilizer and then turn around and resell it on the market for a profit. A market-wide subsidy would not provide this option for the landlord because the price would be reduced for the whole market. Thus, a government subsidy is viable to an extent that a small-scale subsidy by the landlord included in the contract is not.

Additionally, we assume that the government can observe the tenants wealth levels, as we assumed the landlord could. Therefore the government can differentiate the tax on landlords based on the wealth level of their tenants. This assumption follows the general rule that the information available to the government is the same as that available to landlords. The only difference between the government and individual landlords is their ability to institute a large-scale, market-wide subsidy.

To represent the subsidy we will replace the price of fertilizer, p, with a new subsidized price of fertilizer represented by $(1 - s)p$. It is assumed that $0 \leq s \leq 1$. When $s = 0$ there is no subsidy. When $s = 1$ fertilizer is fully subsidized to the point that it is free to tenants. Values in between represent various levels of subsidy.

The subsidy is paid for by a tax on the tenants earnings. As stated above, we assume that all tenants are identical except for their wealth levels, therefore the government can determine the expected level of fertilizer use for a tenant based on the tenant's wealth level. The government differentiates between landlords based on the wealth of their tenants, and taxes landlords the expected cost of the subsidy for their tenant, spX. Where X is the fertilizer choice expected of that tenant, which will be different for tenants of different wealth levels, but identical among tenants of equal wealth.

Because the participation constraint (PC) binds in the case under consideration, the tenant will always be pushed to their reservation income. Thus, the tenants expected earnings will always just be K. Any tax and subsidy scheme will thus not affect the welfare of the tenant. Therefore, if a tax-and-subsidy improves or at least keeps constant the welfare of all landlords it will represent a Pareto-improvement; some individuals will be better off and none will be worse off.

To analyze the effect of the tax-and-subsidy scheme on landlords we use an adapted version of the expression for tenants earnings obtained above. This expression is first adapted by replacing the price, p, with the subsidized price, $(1 - s)p$. Secondly, the tax paid by the landlord is subtracted from the expression. The expression for a tenant's total expected earnings under the tax-and-subsidy scheme is denoted as A. It is shown below:

$$A = Q(L, X) - L - (1 - s)pX - K - spX$$

This simplifies to:

$$A = Q(L, X) - L - pX - K$$

To determine the effect of a subsidy on the landlord's expected earnings we take the derivative of A with respect to the value of the subsidy s. This derivative is evaluated below:

$$\frac{dA}{ds} = Q_X X_S + Q_L L_S - L_S - pX_S$$

Which can be rearranged to obtain:

$$\frac{dA}{ds} = (Q_X - p)X_S + (Q_L - 1)L_S$$

The value of this derivative, and thus the effect of the subsidy will be different for tenants of different wealth levels.

First, we examine the effect of the subsidy for the landlords of rich tenants, those with $W > \overline{W}$. These tenants, as demonstrated above in Case 3 (a), choose the efficient fertilizer and labor levels, that is $Q_X = p$ and $Q_L = 1$. Thus for their landlords the above expression of the derivative becomes just $\frac{dA}{ds} = 0$. The subsidy has no effect on the welfare of these landlords. Tenants were choosing efficient levels of labor and fertilizer and subject to the subsidy they will continue to choose the efficient level, although what the efficient level will be will have changed. Thus contracts will still be efficient and welfare will not change for these landlords or tenants.

Secondly, we examine the effect of the subsidy on landlords of poor tenants, those with $W < \overline{W}$. For these tenants, with a chance of default and thus problems of moral hazard, the efficient levels of fertilizer and labor are not achieved. Specifically these tenants oversupply fertilizer and undersupply labor, that is $Q_X < p$ and $Q_L > 1$. The subsidy increases fertilizer uses because it decreases the cost of fertilizer, so $X_S > 0$ Additionally, the subsidy increase labor input because the increased fertilizer use increases the marginal product of labor, thus $L_S > 0$. These results combined lead us to conclude that the first term of the above derivative will be positive, $(Q_X - p)X_S < 0$, and the second term will be negative, $(Q_L - 1)L_S < 0$. Whether or not the overall effect of the tax-and-subsidy scheme on these landlords expected earnings is positive, whether $\frac{dA}{ds} > 0$, thus depends on the relative magnitudes of the two terms. In other

words, whether or not the tax-and-subsidy scheme will make landlords of poor tenants better off depends on whether the positive effect of inducing additional labor outweighs the negative effect of inducing additional fertilizer use.

We can examine more specifically what conditions must hold for $\frac{dA}{ds} > 0$. We assume $\frac{dA}{ds} > 0$:

$$\frac{dA}{ds} = (Q_X - p)X_S + (Q_L - 1)L_S > 0$$

Rearranging we obtain:

$$(Q_L - 1)L_S > -(Q_X - p)X_S$$

This demonstrates the result described above, for the tax-and-subsidy scheme to have a positive effect the gain in efficiency from the induced additional labor must be greater in magnitude than the loss in efficiency from the additional fertilizer induced.

If the above condition holds then $\frac{dA}{ds} > 0$ for the landlords of poor tenants. Thus, combined with the above results I have demonstrated that if the above condition holds the tax-and-subsidy scheme will lead to a Pareto improvement: landlords of poor tenants will be made better off, landlords of rich tenants will have no change in expected earnings, and all tenants will have no change in expected earnings because they will always be pushed to their reservation income. This represents a Pareto improvement because one group is made better off and no one is made worse off.

By demonstrating that there is a possible Pareto improvement to this equilibrium we have demonstrated that Distribution B will not be even *constrained* Pareto efficient, under certain conditions. Combined with the above result that Distribution A will be first-best Pareto efficient (and therefore also constrained efficient), we have demonstrated that whether or not equilibrium

is constrained Pareto efficient can depend on the distribution of wealth. We have not

demonstrated that it will depend on the distribution in all cases of the world, just that it can under

certain conditions. In contrast, with perfect information whether or not equilibrium is Pareto

efficient would *never* depend on the distribution of wealth. With imperfect information this is no

longer always the case, sometimes distribution affects efficiency. Issues of equity cannot

assumed to be separated from issues of efficiency, the two can be interlinked in certain situations.

For example, in this case a more equitable distribution can lead to an equilibrium that is first-best

efficient, when a less equitable distribution leads to an equilibrium that is not even constrained

efficient.

5. Caveats and Areas for Further Research

There are two main caveats to the conclusions described above. The first applies to both

models. The second only applies to the model with fertilizer. The implications of both could be

explored further in subsequent research.

θ_1's Dependence on α

In Ray and Singh (1998) the authors acknowledge explicitly that θ_1 depends on α. In

other words, the conditions under which a tenant will default depend on the cropshare. This is

clear from the definition of θ_1:

$$\theta_1(\alpha, C, L; W): \begin{cases} \alpha\theta_1 Q(L) + W = C \\ \underline{\theta} \ if \ \alpha\underline{\theta}Q(L) + W \geq C \end{cases}$$

Later in the paper, however, when Ray and Singh obtain the first-order condition for α, they do

not take into account the fact that θ_1 depends on α. Recall, the Lagringian for the landlord's

maximization:

$$F(\lambda, \mu, \alpha, C, L) = \int_{\theta_1}^{\overline{\theta}} [(1-\alpha)\theta Q(L) + C] dF(\theta) + \int_{\underline{\theta}}^{\theta_1} \theta Q(L) + W] dF(\theta) + \lambda$$

$$\cdot \left[\int_{\theta_1}^{\overline{\theta}} [\alpha \, \theta Q(L) - C] dF(\theta) + \int_{\underline{\theta}}^{\theta_1} \theta Q(L) + W] dF(\theta) - K \right] + \mu \cdot (1-\alpha)$$

They and I then take the derivative of this expression to be:

$$F_\alpha(\lambda, \mu, \alpha, C, L) = \int_{\theta_1}^{\overline{\theta}} [-\theta Q(L) + (1-\alpha)\theta Q'(L) L_\alpha] dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta Q'(L) L_\alpha] dF(\theta) +$$

$$\lambda \left[\int_{\theta_1}^{\overline{\theta}} [\theta Q(L) + \alpha Q'(L) L_\alpha] dF(\theta) - L_\alpha \right] - \mu = 0$$

However, this derivative does not take into account the fact that θ_1 depends on α. For simplicity, in this paper I have followed the approach of Ray and Singh and ignored this effect. However, further research should examine if taking this effect into account changes the results of the contracting problem.

The existence of sharecropping in the adapted model

In the preceding section to demonstrate that constrained efficiency could depend on distribution it was useful to assume that Case 3 held in the adapted model as it always did in the unadapted model. However, as mentioned above the addition of fertilizer allows for either Case 2 or Case 3 to hold, depending of certain conditions. In other words, in the adapted model with fertilizer solutions to the contracting problem can exist where $\alpha < 1$, sharecropping can arise in this model. This is a very interesting result and further research into the conditions that give rise to the two cases and the implications of an additional input leading to sharecropping could be very fruitful. To some extent an in-depth investigation of these issues would be outside the primary scope of this paper. However, a brief examination of the two cases and the conditions

conditions under which they would arise follows. As stated above, and demonstrated in an appendix Case 1 will not arise in this model, just as in the unadpated model.

Case 2: participation constraint binds, share constraint does not bind

For this case we assume that the participation constraint (PC) binds and the share constraint (SC) does not bind, that is the tenant is pushed to their reservation income and sharecropping is present in the model. Formally, we assume $\alpha < 1$, and $\mu = 0$. Unlike, the Ray and Singh model without fertilizer we find that this case can arise under certain conditions.

Given that the participation constraint (PC) binds we have:

$$\int_{\theta_1}^{\overline{\theta}} [\alpha\theta Q(L,X) - C]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} (-W + pX)dF(\theta) - L - pX = K$$

It is clear that if $\theta_1 = \overline{\theta}$, (PC) would become $-W - L = K$. Because we assume $K > 0$, we would either have that the tenant is indebted or the above expression would be a contradiction. Either way we can conclude that for this case to be a solution to the contracting problem $\theta_1 < \overline{\theta}$. Now we can examine the first order condition for C:

$$F_C(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\overline{\theta}} dF(\theta) - \lambda \int_{\theta_1}^{\overline{\theta}} dF(\theta) = 0$$

Which we can rearrange to:

$$\int_{\theta_1}^{\overline{\theta}} dF(\theta)\,(1 - \lambda) = 0$$

Given that $\theta_1 < \overline{\theta}$, we can conclude that $\int_{\theta_1}^{\overline{\theta}} dF(\theta) > 0$, and thus that $\lambda = 1$. Therefore the first order condition for α becomes:

$$F_\alpha(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\bar{\theta}} [-\theta Q(L, X) + (1 - \alpha)\theta(Q_L L_\alpha + Q_X X_\alpha)]$$

$$+ \int_{\underline{\theta}}^{\theta_1} [\theta(Q_L L_\alpha + Q_X X_\alpha) - pX_\alpha]dF(\theta)$$

$$+ \int_{\theta_1}^{\bar{\theta}} [\theta Q(L, X) + \alpha\theta(Q_L L_\alpha + Q_X X_\alpha)]dF(\theta) + \int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta) - L_\alpha - pX_\alpha = 0$$

Which simplifies to:

$$(Q_L L_\alpha + Q_X X_\alpha)\left(1 - \alpha \int_{\theta_1}^{\bar{\theta}} \theta dF(\theta)\right) - \int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta) + L_\alpha\left(\int_{\theta_1}^{\bar{\theta}} \alpha\theta Q_L dF(\theta) - 1\right)$$

$$+ X_\alpha\left[\int_{\theta_1}^{\bar{\theta}} (\alpha\theta Q_X - p)dF(\theta)\right] = 0$$

Then using ICX and ICL we can determine that the last two terms are equal to zero and drop out.

The above expression then becomes:

$$(Q_L L_\alpha + Q_X X_\alpha)\left(1 - \alpha \int_{\theta_1}^{\bar{\theta}} \theta dF(\theta)\right) - \int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta) = 0$$

Solving for α the above expression becomes:

$$\alpha = \frac{1 - \dfrac{\int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta)}{Q_L L_\alpha + Q_X X_\alpha}}{\int_{\theta_1}^{\bar{\theta}} \theta dF(\theta)}$$

We assume for this case $\alpha < 1$. Thus for this case to arise as a solution to the contracting

problem the above expression must be greater than 1. This case will therefore hold if:

$$\frac{1 - \dfrac{\int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta)}{Q_L L_\alpha + Q_X X_\alpha}}{\int_{\theta_1}^{\bar{\theta}} \theta dF(\theta)} < 1$$

Which can be rearranged to obtain:

$$1 - \frac{\int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta)}{Q_L L_\alpha + Q_X X_\alpha} < \int_{\theta_1}^{\overline{\theta}} \theta dF(\theta)$$

Thus this case will hold as a solution to the contracting problem, and thus sharecropping will arise, if the above expression holds. Whether or not this expression holds will depend on the price of fertilizer and the specific form of the production function. Further examination of when this case will arise and the intuition behind why it arises in my adapted model and not in Ray and Singh's model is left to later research. In short though, I surmise that the difference arise because of the odd "double moral hazard" effect on fertilizer which is unique to my model.

Case 3: participation constraint binds, cropshare constraint binds

For this case we assume that the participation constraint (PC) binds and the cropshare constraint (CC) binds. Formally, we assume that $\lambda \geq 0$, $\mu \geq 0$, and $\alpha = 1$. I will demonstrate that unlike in the Ray and Singh model, where this case would always hold, this case will only arise under certain conditions.

Following the same logic used in Case 2 we can conclude from the first-order condition for C that $\lambda = 1$.

Given that $\alpha = 1$ and $\lambda = 1$, the first-order condition for α becomes:

$$F_\alpha(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\overline{\theta}} -\theta Q(L, X) dF(\theta) + \int_{\underline{\theta}}^{\theta_1} [\theta(Q_L L_\alpha + Q_X X_\alpha) - pX_\alpha] dF(\theta)$$

$$+ \int_{\theta_1}^{\overline{\theta}} [\theta Q(L, X) + \theta(Q_L L_\alpha + Q_X X_\alpha)] dF(\theta) + \int_{\underline{\theta}}^{\theta_1} pX_\alpha dF(\theta) - L_\alpha - pX_\alpha - \mu$$

$$= 0$$

Which with some cancellation and rearranging becomes:

$$\int_{\underline{\theta}}^{\theta_1} [\theta(Q_L L_\alpha + Q_X X_\alpha) - pX_\alpha]dF(\theta) + L_\alpha \left(\int_{\theta_1}^{\overline{\theta}} \theta Q_L dF(\theta) - 1 \right) + X_\alpha \left[\int_{\theta_1}^{\overline{\theta}} (\theta Q_X - p)dF(\theta) \right]$$

$$= \mu$$

Given that $\alpha = 1$, ICL gives us the result that $\int_{\theta_1}^{\overline{\theta}} \theta Q_L dF(\theta) - 1 = 0$, so the second term

drops out. Similarly, given that $\alpha = 1$, ICX can be rearranged to obtain $\int_{\theta_1}^{\overline{\theta}} (\theta Q_X - p)dF(\theta) = 0$,

so the last term drops out as well. Thus, we obtain:

$$\int_{\underline{\theta}}^{\theta_1} [\theta(Q_L L_\alpha + Q_X X_\alpha) - pX_\alpha]dF(\theta) = \mu$$

For this case to hold we must have $\mu \geq 0$. Thus, this case will hold as a solution to the

contracting problem if the following expression holds:

$$\int_{\underline{\theta}}^{\theta_1} [\theta(Q_L L_\alpha + Q_X X_\alpha) - pX_\alpha]dF(\theta) \geq 0$$

Again whether or not this case holds depends on the price of fertilizer and specific of the

production function, as well as the distribution of θ. As before a more in-depth analysis of why

and when this case will hold is left to subsequent research.

Overall, we have demonstrated that with the introduction of fertilizer it is no longer clear

whether sharecropping will or will not arise in the model. This is an interesting result and would

be a viable topic for further research. Additionally, it would be interesting to study the welfare

properties of contracts when sharecropping arises and compare them to the above results for

Case 3 when sharecropping does not arise.

6. Conclusion

Stiglitz and others working in the economics of information have emphasized the many

ways that introducing imperfect information into neo-classical economic models changes the

basic conclusions of those models. One of those basic conclusions was that issues of equity and efficiency could be separated: with perfect information the Pareto efficiency of equilibrium did not depend on the initial distribution of wealth. But as Stiglitz (1987, 1985) pointed out, when imperfect information is introduced the initial distribution of wealth can affect whether or not the equilibrium is Pareto efficient. Thus, concerns about equity and concerns about efficiency cannot be separated when imperfect information is taken into account; the two concerns are interrelated.

One of the reasons that distribution affects efficiency when imperfect information is introduced is that the distribution can affect whether or not transactions that include moral hazard will occur. For example, as Stiglitz (1991) writes, "the incentives problems associated with sharecropping would not arise if all tenants owned the land upon which they worked." Sharecropping, an institution common in developing countries, is often used as an example of moral hazard in general, as well as the way distribution can affect whether or not moral hazard arises. Rental contracts with limited liability are another model of land tenure in developing countries. Similar to sharecropping, land tenure contracts with limited liability lead to moral hazard and whether or not that moral hazard will arise depends on the distribution of wealth.

Thus in this paper I have used a model of land tenure with limited liability (Ray and Singh 2001) to demonstrate an example of the effect described by Stiglitz: that with imperfect information whether or not the equilibrium of an economy is Pareto efficient can depend on the distribution of wealth because the existence of moral hazard can depend on distribution. Thus, I demonstrate examples where one distribution of wealth leads to an equilibrium that is Pareto efficient and another distribution of wealth leads to an equilibrium that is not Pareto efficient. I have demonstrated this effect in two slightly different ways, using two slightly different models to do so. First, I demonstrated using the Ray and Singh (2001) model that distribution can affect

whether or not equilibrium was first best efficient. Then adapting their model slightly I was able to demonstrate that under certain conditions the distribution could affect whether or not the

While this example was very specific, the conditions had to be just so to allow for one distribution to be first-best Pareto efficient and another to be not even constrained efficient, it demonstrated that when imperfect information is present distribution can affect efficiency in some situations. This is not a trivial result: in models with perfect information the distribution of wealth will never affect the Pareto efficiency of equilibrium. The separation of equity and efficiency is one in the growing list of conclusions that could be taken for granted when perfect information was present but that fail to hold in all cases when imperfect information is taken into account.

Imperfect information, such as asymmetric information about labor effort between a landlord and his tenant, is rampant in the real world. Examining, the specific ways that its existence changes the conclusions of economic models thus seems like a worthwhile pursuit. By demonstrating an example of one such change, described by Stiglitz but not demonstrated formally, I hope to have added to that pursuit. Specifically, I have demonstrated that when imperfect information is present, as it generally is in the real world, the separation of equity and efficiency cannot be taken for granted.

Appendix: Examination of Case 1 in adapted model

Case 1: participation constraint does not bind

      As above, in this case we assume that the participation constrain (PC) does not bind. As before, we will find that this case does not arise as a solution to the contracting problem. Given that (PC) does not bind $\lambda = 0$, and thus the first order condition for C becomes:

$$F_C(\lambda, \mu, \alpha, C) = \int_{\theta_1}^{\overline{\theta}} dF(\theta) = 0$$

From this we can conclude that $\theta_1 = \overline{\theta}$. In other words, the tenant is so poor that they default under all conditions. The maximization of the tenant's expected earnings now becomes:

$$\int_{\underline{\theta}}^{\overline{\theta}} [\alpha\theta Q(L, X) - C]dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} (-W + pX)dF(\theta) - L - pX$$

Which simplifies to just:

$$-W - L$$

      Again, we have the result that $L^* = 0$, that is at equilibrium the tenant chooses to not put out any labor because they know they will default under all conditions. The value of X^* is unclear in this case. That is, it is unclear how much fertilizer the tenant will choose to purchase and use, because while he or she will not gain at all from increases in output due to additional fertilizer, it will also not matter to the tenant how much they spend on fertilizer because they will always default and lose all their wealth anyway. However, because we still assume that $Q(L^*) = 0$, that is if a tenant puts out no labor (regardless of how much fertilizer they purchase and use) they will get no output, we get the result that $W = C$. The fixed-rent is equal to the tenant's wealth level. Then from the participation constraint (PC) we find that $W < -K$, that is the tenant is indebted.

Given that $\theta_1 = \bar{\theta}$, the landlords expected earnings becomes just:

$$W - pX$$

Because $pX \geq 0$, we can conclude that $W - pX < -K$. We see as before that the

landlords expected earnings is less than negative the tenants reservation income. Again, the

tenant would be better off not leasing the land at all and just leaving it fallow because in that case

they would have expected earnings of 0, which would be greater than –K and thus greater than

their expected earnings by renting under this case. Therefore, even with the addition of fertilizer,

this case will not arise as a solution to the contracting problem, because the landlord would be

better off not renting at all.

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