Behavioral Growth Theory: A Neoclassical Approach

Munik K Shrestha*

Department of Economics, Haverford College

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Abstract

How do material norms and aspirations on which individual judgments of well-being are based affect the growth dynamics of an economy? Do economies that weigh very highly of these norms grow differently from those economies that values these norms differently? This paper attempts to answer these questions by building a growth model, where individual not only care for its consumption but also evaluates its consumption with its perceived level of ideal consumption. We show that it is best for the economy that individuals ignore material aspirations when the economy is growing, whereas they should highly care about their material aspiration when the economy is depleting. We also show that role of individual’s outlook is underestimated in contemporary growth theories.

Thesis Advisors

Richard Ball
Associate Professor, Department of Economics

Indradeep Ghosh
Assistant Professor, Department of Economics

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*To whom correspondence should be addressed: Email author: mshresth@haverford.edu
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1 Introduction and Background

A simple question used by Hadley Cantril (1965) for empirical studies on happiness may convey the motivation for this study.

Here is a picture of a ladder. Suppose we say that the top of the ladder (pointing) represents the best possible life for you and the bottom (pointing) represents the worst possible life for you.

(C) Where on the ladder (moving finger rapidly up and down ladder) do you feel you personally stand at the present time? Step number [    ]

Provided this, consider an economy $A$ that consists of identical individuals with equal income $\$30$ thousand and with the ladder-step number 3 (out of 10), and an economy $B$ identical to economy $A$ in every respect except that individuals in economy $B$ has ladder-step number 9. Since economy $A$ differs with economy $B$ only on ladder-step number, one can interpret this scale as if lower value of ladder-scale corresponds to a strong preference and value towards higher level of perceived ideal income or consumption. In other words, economy $A$ can be thought of as individuals with stronger weight for higher perceived ideal income than that of economy $B$. From this interpretation what we can say is that individuals care not only for their absolute income, but also for their relative income with respect to their perceived ideal income.

Given this consideration and interpretation, one might contemplate if economy $A$ grows differently from economy $B$. According to neoclassical growth model widely used in contemporary economics literature, economy $A$ grows no differently from economy $B$, for neoclassical growth model is independent of individual’s psychological outlook. In contrast, this paper will answer it quite differently by building a model that incorporates individual’s social outlook—thus the title “Behavioral Growth Theory”. In so doing, we consider that individual’s utility depends on both their consumption and their consumption relative to their perceived ideal consumption. Doing this will provide us with tools to analyze how economy $A$ differs from economy $B$ which is different only on their weight on perceived ideal income. We can generally interpret this utility as comparison utility, as individual utility depends both on absolute and relative consumption.

Comparison utility in economics is not a new idea. Clark and Oswald (1998) studied individuals behavior in economic and social settings using comparison concave and convex utility. Although the study was done to examine the effects of individual’s relative position, they showed the dependence of individual’s action on the curvature of its comparison utility function. They concluded that individuals with comparison concave utility follow others action, whereas with comparison-convex utility they act opposite of others.

Some previous studies have been done in growth theory that encompasses individuals evaluation of absolute and some relative consumption. Caroll and et al (1997) uses a comparison utility in two separate models (both based on AK framework) where the reference for relative consumption is given in one model by the average consumption and in the other model by individuals own past consumptions. Following the spirit of Caroll and et al, this paper in similar fashion attempts to model a micro-founded growth theory with a comparison utility, where the reference for relative consumption is given by a perceived ideal consumption as a result of social norms and constructs in the economy. So, using this comparison utility in a growth model,

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1. Prior to the onset of our model in section 2, where savings plays an important role in growth, we may interpret individual’s consumption and income interchangeably for simplicity, as higher income normally implies higher consumption.

2. From now on, we will use (until explicitly mentioned otherwise) the term relative consumption to mean individual’s consumption relative to the perceived ideal consumption.
we can explain some growth differences in economy A and economy B, where perceived ideal consumptions are different. Additionally, perceived ideal consumptions can vary cross-culturally. Thus, the model in this paper serves as an excellent framework for studying cross-cultural growth studies.

Unlike Caroll (1997) however, this model endogenizes market return to investment using nicely behaving production function with labor-augmenting technology and generalizes the reference for relative consumption. Specifics of the production function will be discussed more in section 2 when we build the model. But it is a good place to know that we will consider a model with balanced growth path for steady-steady state equilibrium and thus analyze how this growth path at steady state differs for economy A and economy B.

In what follows from this paper, we will see that economics that care more for relative consumption are actually less well off than those that care relative consumption to lesser extent. To this extent, we can tentatively conclude that economy B will be better off than economy A. In order to do so, our model assumes that perceived ideal consumptions grows at the same rate as actual consumptions. This assumption is affirmed by various studies in economics and psychology (Cantril, 1965; Rainwater, 1990; Easterlin, 1995). In particular Rainwater (1990) concludes that between the period of 1950 and 1986, ideal perceived income grew at the same rate as actual per capita income. Easterlin and Crimmins (1991) also came up with the same conclusion that [perceived ideal income] grew not only in the same direction, but also at the same rate. Similarly, Brickman and Campbell (1971) coined the term “Hedonic Treadmill” to convey their view that with the more possession and accomplishments individuals posses, the more they need to boost their level of happiness.

Easterlin (2003) also points out the difference between complete adaptation and [partial] adaptation of individuals’ material aspiration to changes in circumstances in their lives.

Complete adaptation means that material aspiration increase commensurately with income, and, as result, one gets no nearer to or farther away from the attainment of one’s material goals, and well being is unchanged. [Partial] adaptation means that aspirations change less than the actual change in one’s circumstances.

So, although one might be apprehensive towards using compete adaptation version of one’s perceived ideal consumption to the changes to its actual consumption, we should be careful not to confuse changes in one’s satisfaction or well-being with changes in the actual consumption. Changes in actual consumptions are not equivalent to changes in individual’s well-being. As a matter of fact, changes in one’s well-being can result from changes in actual consumption on the one hand and changes in its relative consumption on the other hand. The reason is that the utility we consider is a comparison utility, where individuals gain satisfaction from both actual consumption and relative consumption. Changing the parameters in utility changes the relative weight for actual consumption and relative consumption. We will see that only when an individual care about relative consumption and not its actual consumption does its change in actual consumption correspond to the change in its well-being at the same rate. Thus in general case, comparison utility incorporates some version of partial adaptation of well-being with changes in actual consumption even when perceived ideal consumptions grow at the same rate as actual consumptions.

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3As a matter of fact, we will assume that perceived ideal consumption grows with technology. So from balanced growth theory for an economy with labor augmenting technology, we know that consumption grows with technology at steady-state. It is in this view we consider that perceived ideal consumption grows with consumption in steady-state.

4We can consider that utility measures some degree of well being from consumption.

5Note that we refer to steady-state because in the model we will assume perceived ideal consumption to grow at the same
What remains to be said is that, for a particular case in the model, where individual only cares for relative consumption, the economy completely assumes Easterlin’s Paradox\(^6\). This is because any changes in actual consumption are offsetted by changes in perceived ideal income, leaving the utility of an individual invariant with changes in actual consumption. We can call this an Easterlin’s Economy. We will show in the paper that individuals in this economy are worst\(^7\) off in the steady state.

Having said that, motivation for this research is to build a framework for understanding cross-cultural growth differences. On the one hand, some culture might possess complete adaptation of people’s perceived ideal income to the changes in actual income. On the other hand, some culture with different material norms and aspirations might possess some version of partial adaptation or no adaptation of their perceived income to the changes in actual income. Given this, this model provides an excellent framework for understanding the growth differences in their respective economies.

In section 2 of the paper we formulate our theoretical model of behavioral growth theory by using the tools of contemporary neoclassical growth model. In section 3, we we will derive an equilibrium sequence of growth and study the model qualitatively. In section 4, we parametrize the model to facilitate simulation and study the model with figures and plots. Before beginning our model, we start with the assumptions of neoclassical model.

1.1 Preliminaries of the Model

The framework for this paper is the standard neoclassical growth model. This section thus involves a brief exposition of assumptions of the neoclassical growth model that we will use in our model. Neoclassical model provides a useful framework to study macroeconomic effects from individual or household preferences and decisions, as one of the thing it is based on is the assumption about household optimization and represents the demand side of the economy fairly well.

Assumption 1: This assumption defines the characteristics (continuity, differentiability, diminishing marginal products) of the production function. Two inputs, capital \((K)\) and labor \((L)\) are used to produce a single, homogenous consumption good.

The production function \(F : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+\) is strictly increasing and twice differentiable in \(K\) and \(L\), and satisfies

\[
F_K(K, L) = \frac{\partial F(K, L)}{\partial K} > 0, \quad F_L(K, L) = \frac{\partial F(K, L)}{\partial L} > 0, \\
F_{KK}(K, L) = \frac{\partial^2 F(K, L)}{\partial K^2} < 0, \quad F_{LL}(K, L) = \frac{\partial^2 F(K, L)}{\partial L^2} < 0
\]

Assumption 2: Constant Return To Scale (C.R.S) Given that production function is homogeneous of degree 1 in its inputs \(K\) and \(L\), proportional change in both of its inputs \(K\) and \(L\) induces equal proportional changes in output \(F\). The main feature of constant return to scale is that it enables us to work with per capita production function, which is crucial in the model.

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\(^6\)Easterlin’s Paradox implies that happiness is not linked with economic growth.

\(^7\)We will also consider some exceptions where Easterlin’s economy enjoys relatively high consumption and capital level in steady-state.
Assumption 3: Neoclassical Preference  This assumption characterizes the necessary condition for individuals preferences in order to facilitate optimization problem. The utility function $U(c)$, where $U : \mathbb{R}^+ \rightarrow \mathbb{R}$ is strictly increasing, concave and twice differentiable in $c$ and $U_c > 0$ and $U_{cc} < 0$, $\forall c \in \mathbb{R}^+$.

Inada Condition: Production function $F$ satisfies the Inada conditions which is
\[
\lim_{K \to 0} F(K, L) = \infty, \quad \lim_{K \to \infty} F(K, L) = 0 \text{ for all } L > 0,
\]
\[
\lim_{L \to 0} F(K, L) = \infty, \quad \lim_{L \to \infty} F(K, L) = 0 \text{ for all } K > 0, \text{ and}
\]
\[
F(0, L) = 0 \text{ for all } L,
\]

Representative Household and Firm  Note that we can use the term household, agents and individual interchangeably. So to confirm with the widely used term in growth theory, we use the term household for individual. When the economy admits representative household and representative firm, the demand and supply side of the economy can be respectively represented by a single household and a firm as if all the decisions about savings and consumptions are made by the single household and the firm. The utility of this notation is that rather than modeling the demand side of the economy with many heterogeneous households, we model with the single representative household. However, in our model the economy automatically admits representative household households are assumed to be identical. The economy also admits representative firm as long as there is no externalities in production and all factors of production are priced competitively. Our model assumes no production externalities all factors are priced competitively; hence we can use representative firm in our model.

2 The Model

We consider an infinite-horizon economy with identical households in continuous time. Since households are identical, the economy automatically admits the representative household. Instantaneous utility for the representative household is given by
\[
U(c(t), d(t)) = \left( \left( \frac{d(t)}{c(t)} \right)^{\alpha} c(t)^{1-\alpha} \right)^{1-\theta},
\]
where $c$ refers to the household’s actual consumption at time $t$; $d$ is some exogenously given level of the household’s perceived ideal consumption, which is assumed to be higher than its actual consumption. The household’s utility (1) satisfies Assumption 3— it is increasing, concave and twice differentiable in $c$ for $\theta < 1$. Given this functional form of utility, the household derives utility not only from its actual consumption but also from its relative consumption, where the reference for relative consumption is provided by the household’s perceived ideal consumption. This will be clear when we notice that (1) can be written as
\[
U(c(t), d(t)) = \left( \phi^\alpha(t) c(t)^{1-\alpha} \right)^{1-\theta},
\]
where
\[
\phi(c(t), d(t)) = \frac{c(t)}{d(t)}
\]
\[8\text{We normally call this representative household and representative firm.}\]
Here, $\phi$ can be interpreted as a comparison utility, where the household\(^9\) can gain utility from its increased level of actual consumption relative to ideal consumption level $d$. Leaving $d$ unchanged at a particular time, higher household consumption thus increases the household’s utility. As a matter of fact, the household’s utility increases with actual consumption $c$ through two ways: one is just from its higher actual consumption and the other is from the increase in household’s comparison utility. This can also be understood mathematically. At a particular time $t$, the derivative of the utility $U$ with $c$ is given by

$$\frac{dU(c, \phi)}{d(c)} = \frac{\partial U}{\partial (\phi)} \frac{d\phi}{d(c)} + \frac{\partial U}{\partial (c)}$$

(4)

From equation (1), (2), and (3) respectively, note that

$$\frac{\partial U}{\partial (c)} > 0, \quad \frac{\partial U}{\partial (\phi)} > 0, \quad \frac{d\phi}{d(c)} > 0$$

(5)

Therefore, when actual consumption $c$ increases, household utility $U$ increases via increment in its actual consumption $\frac{dU}{d(c)}$ alone and increment in comparison utility $\phi$ by $\frac{d\phi}{d(c)} > 0$. Additionally, when ideal household consumption $d$ changes without any change in actual consumption $c$, we note that the household’s utility $U$ decreases because (by chain rule)

$$\frac{dU(d)}{d(d)} = \frac{dU}{d(\phi)} \cdot \frac{d\phi}{d(d)}$$

(6)

and $\frac{dU}{d(\phi)} > 0$ and $\frac{d\phi}{d(d)} < 0$.

Parameters of the utility $U$ in equation (1) plays an important role for analysis. The parameter $\alpha \in [0, 1]$ should be interpreted as weight the household utility function places on relative consumption and actual consumption. Higher $\alpha$ reflects higher importance to comparison or relative consumption. As a matter of fact, when $\alpha$ is 1, the household only care about relative consumption. When $\alpha$ is 0, households only consider actual consumption and ignores relative consumption. When $\alpha$ is between 0 and 1, both actual and relative consumption are considered. For example, for $\alpha = \frac{1}{2}$, a household with 2 units of actual consumption $c$ and 4 units of ideal consumption $d$ have same utility, as the household with 1 unit of both absolute and desired consumption.

The population of households in the economy is assumed constant, and the population is $L$ for all time. For the representative household, the objective function at the beginning of the period is given by

$$\int_0^\infty U(c(t), d)exp(-\rho)dt,$$

(7)

where $\rho$ refers to the discounting of future utility streams.

**Production Function** We consider a labor augmenting production function, which is also referred to as Harrod-neutral technology production. The production function is given by

$$Y(t) = F(K(t), A(t)L),$$

(8)

\(^9\)We will now just use the term household while referring to the representative household.
where technological progress is

\[ A(t) = A(0) \exp(gt) \]

\[ \dot{A}(t) = g A(t) \]  

This production function satisfies Assumption (1), Inada conditions, and CRS with respect to its inputs \( K(t) \) and \( L \). It is important to note that (8) is purely labor-augmenting (Harrod-neutral) technology production. By Uzawa’s theorem, we can primarily conclude that this production function will achieve balanced growth. Because of constant return to scale, we write production function in terms of average production per effective capita.

\[ y(t) = \frac{Y(t)}{A(t)L} = F \left( \frac{K(t)}{A(t)L}, 1 \right) = f(k(t)), \quad (10) \]

where

\[ k(t) = \frac{K(t)}{A(t)L} \]

(11)

is \( k(t) \) the effective capital per labor ratio or the average effective capital.

**Market Clearing Condition**  
We consider a competitive economy, where households own the capital stock and rent it to firms at a rental price \( R(t) \) at time \( t \). In the market clearing condition (MCC), the total supply of capital from households is exactly equal to the total demand of capital by firms. In other words,

\[ K(t) = \bar{K}(t) \]

Households also supply labor inelastically at a rental labor price or wage rate of \( w(t) \). Inelasticity of labor supply means that all labors in the economy are employed at a wage rate \( w(t) \), given that wage rate always stays positive. So in the market clearing condition,

\[ L(t) = \bar{L}(t) \]

It is with this understanding that whenever we write \( K(t) \) and \( L(t) \) from now on in this paper, we will be referring to market clearing conditions for capital and labor, where above two equations for the MCC are satisfied.

**2.1 Firms Optimization**

Given the market clearing condition with technology \( A(t) \) and factor prices \( R(t) \) and \( w(t) \), firms choose to maximize its profit, where profit at a time \( t \) is given by

\[ F(K(t), A(t)L(t)) - R(t)K - w(t)L(t) \]

So the profit maximization problem for the representative firm at time \( t \) is given by

\[ \max_{K \geq 0, L \geq 0} F(K, A(t)L) - R(t)K - w(t)L \]  

(12)
Since the production function $F(K, A(t)L)$ is concave, the optimization problem is concave. Hence, solving for the optimization problem, the optimal rental rate of capital and wage rate is

$$R(t) = F_K(K(t), A(t)L) = f'(k(t))$$ (13)

and

$$w(t) = F_L(K(t), A(t)L) = f(k(t)) - k(t)f'(k(t))$$ (14)

Since capital depreciates at a rate $\delta$, the market return to the capital $r(t)$ is thus given by the relation

$$r(t) = R(t) - \delta = f'(k(t)) - \delta$$ (15)

**Equation of Motion:** We consider that households only choose capital for investment and there is no government bond in the economy. So when market clears, assets per capita must be equal to capital stock per capita. Since from the demand side, each household optimizes consumption (next section), the law of the motion of capital or asset of representative household is given by

$$\dot{k}(t) = (r(t) - g)k(t) + w(t) - \bar{c}(t),$$ (16)

where $k(t)$ is the effective capital per labor ratio, $g = \frac{A(t)}{A(t)}$ the growth rate of technology, and $\bar{c}(t)$ is the effective consumption per capita which is given by

$$\bar{c}(t) = \frac{c(t)}{A(t)L}$$

where $c$ is the average consumption, $c(t) = \frac{C(t)}{A(t)}$.

Using equation equation (14) in equation (16), we can also write equation of motion as

$$\dot{k}(t) = f(k(t)) - (g + \delta)k(t) - \bar{c}(t)$$ (17)

### 2.2 Households Optimization

Households optimization problem becomes such that households maximizes its streams of utility, given by objective function (7)

$$\int_0^{\infty} U(c(t), d)exp(-\rho)dt$$

subject to the resource (capital) constraint (17)

$$\dot{k}(t) = (r(t) - g)k(t) + w(t) - \bar{c}(t)$$

and to transversality condition such that household does not asymptotically acquire negative wealth. This transversality for an infinitively lived household can be written mathematically as

$$\lim_{t \to \infty} \left[ k(t)exp\left(-\int_0^t (r(s) - g)ds\right) \right] \geq 0$$ (18)
Setting up the optimization problem using dynamic optimization theory, the current value Hamiltonian becomes

\[ H = U(c(t), d(t)) + \lambda(t)(w(t) + r(t)k(t) - c(t)), \] (19)

where \( \lambda \) is a current-value co-state variable, \( c \) is a control variable and \( k \) is the state variable. Note that the current value Hamiltonian is concave with respect to the control variable \( c \), as the law of motion is linear in \( c \) and the utility function \( U \) is concave in \( c \). Hence, we can use methods from dynamic optimization to find the optimal consumption path. Since the household live for an infinite period, transversality condition (18) for the optimal path can also be written as

\[ \lim_{t \to \infty} \left[ \exp\left(\left(\rho - g\right)t\right) \lambda(t) \ k(t) \right] = 0, \] (20)

This transversality condition for the optimal path which is written in terms of effective shadow price \( \lambda \) means that the discounted current-value effective price of the capital (asset) converge to zero as the discounted time period goes to infinity. We will show at the end of this section using results from household optimization problem that this transversality condition (20) for optimal path does satisfy transversality condition for any feasible paths of \((k(t), c(t))\) given by equation (18)

The optimality conditions for the representative household are given by

\[ \frac{\partial H}{\partial c} = U_c(t) - \lambda(t) = 0 \] (21)

and

\[ \frac{\partial H}{\partial k} = \lambda(t)r(t) = -\dot{\lambda}(t) + \rho\lambda(t) \] (22)

or

\[ \frac{\dot{\lambda}(t)}{\lambda(t)} = -(r(t) - \rho) \] (23)

Note from equation (21) that \( U_c(t) = \lambda(t) \), so

\[ \dot{U}_c(t) = \dot{\lambda}(t), \] (24)

Thus, from (21), (22), and (24) we get

\[ \frac{\dot{U}_c(t)}{U_c(t)} = \frac{\dot{\lambda}}{\lambda} = -(r(t) - \rho), \] (25)

Also, since \( U_c(t) \) and \( c(t) \) is continuously differentiable in time , we note that:

\[ \ddot{U}_c = U_{cc}\dot{c} + U_{cz}\dot{d} \] (26)

Substituting this in equation (25), we get

\[ \frac{U_{cc}\dot{c} + U_{cz}\dot{d}}{U_c} = \frac{\dot{\lambda}}{\lambda} = -(r(t) - \rho), \] (27)

Now from household utility function,
\[
\frac{U_{cc}}{U_c} = -\frac{\theta}{c(t)}
\]  

(28)

and

\[
\frac{U_{cd}}{U_c} = \frac{\alpha(\theta - 1)}{d(t)}
\]

(29)

Placing these results in equation (27) we get,

\[-\frac{\dot{c}(t)\theta}{c(t)} + \frac{\dot{d}(t)\alpha(\theta - 1)}{d(t)} = \frac{\lambda}{A} = -(r(t) - \rho),\]

(30)

giving us the growth rate of consumption:

\[
\frac{\dot{c}}{c} = \frac{1}{\theta} \left( r(t) - \rho - \alpha(1 - \theta) \frac{\dot{d}}{d} \right)
\]

(31)

This together with equation of motion given by (17) i.e

\[
\dot{k}(t) = f(k(t)) - (g + \delta)k(t) - \tilde{c}(t),
\]

leaves us with two required equations to study the growth dynamics. Note that equation (31) is a growth rate of average consumption \(c(t)\) as opposed to the growth rate of average effective consumption \(\tilde{c}(t)\) since it came directly from household utility function (1) through household optimization problem. Equation of motion (17) however does contain per capita effective consumption \(\tilde{c}(t)\).

2.3 Equilibrium Characterization

The equilibrium pathway will be the one that consists of paths very similar to competitive equilibrium of the neoclassical growth model, where the representative household maximizes its streams of utility (7) subject to capital constraint (17) and where the path consists of per effective capita consumption, average effective capital, wage and rental rates, such that the factor prices are given by firms optimizing conditions (15) and (14). In addition to this, the steady-state equilibrium point is when rate of change of effective per capital \((\dot{k})\) and effective per capita consumption \((\dot{\tilde{c}})\) is 0.

Note that \(\frac{d\tilde{c}}{dt}\) can be written as,

\[
\frac{d\tilde{c}(t)}{dt} = \frac{\dot{c}(t)A(t) - \dot{A}(t)c(t)}{A(t)^2} = \left( \frac{\dot{c}(t)}{c(t)} - \frac{\dot{A}(t)}{A(t)} \right) \frac{c(t)}{A(t)}
\]

Hence,

\[
\frac{d\tilde{c}(t)}{\tilde{c}} = \frac{\dot{c}(t)}{c(t)} - \frac{\dot{A}(t)}{A(t)} = \frac{\dot{c}(t)}{c(t)} - g
\]

Thus at steady-state equilibrium,

\[
\frac{\dot{c}(t)}{c(t)} = g
\]

(32)
Motion of ideal consumption ($d$) Our assumption in the model is that it grows with technology. With a progress in technology, new wants and consumption desires are created, and hence the desire for consumption grows with technological progress. So, we consider the motion of $d$ by

$$\frac{\dot{d}(t)}{d(t)} = \frac{\dot{A}(t)}{A(t)} = g$$

(33)

In steady-state equilibrium condition, equations (32) and (33) tells us that

$$\frac{\dot{d}(t)}{d(t)} = \frac{\dot{c}(t)}{c(t)}$$

(34)

Note this condition (34) for equilibrium can also be interpreted as if the ratio of the actual consumption to the ideal consumption ($c(t) / d(t)$) remains constant at the steady-state level. We will show this by taking the time derivative of the ratio $c(t) / d(t)$ using equation (34).

$$\frac{d}{dt} \left( \frac{c(t)}{d(t)} \right) = \frac{\dot{c}(t)d(t) - \dot{c}(t)d(t)}{d(t)^2}$$

(35)

So

$$\frac{d}{dt} \left( \frac{c(t)}{d(t)} \right) = \frac{\dot{c}(t)}{d(t)} - \frac{\dot{c}(t)d(t)}{d(t)^2}$$

(36)

In steady-state, we know that

$$\frac{\dot{c}(t)}{c(t)} = g \quad \& \quad \frac{\dot{d}(t)}{d(t)} = g$$

(37)

Thus,

$$\dot{c}(t) = \dot{c}(t)g \quad \& \quad \dot{d}(t) = \dot{d}(t)g$$

(38)

Plugging these results in equation (36) we get

$$\frac{d}{dt} \left( \frac{c(t)}{d(t)} \right) = \frac{\dot{c}(t)g}{d(t)} - \frac{\dot{c}(t)g}{d(t)} = 0$$

(39)

Hence at steady-state, the ratio of the household’s absolute consumption to desired consumption is constant\textsuperscript{10}.

2.4 Transversality Condition

Transversality condition plays a same role as boundary conditions play in a simple ordinary differential equation. Without transversality condition, household optimization problem with maximizing equation (7) subject to (17) gives us many feasible paths for ($k(t), c(t)$), for this model considers an infinitely lived household, which leaves us with free termination boundary condition. But following the theorem for “Maximum Principle for Discounted Infinite-Horizon Problem” (Acemoglu, 2009, p.g 254), transversality condition given

\textsuperscript{10}As discussed in Section 1.
by equation (20) i.e
\[
\lim_{t \to \infty} \left[ \exp(-\rho g t) \lambda(t) k(t) \right] = 0,
\]
provides us the unique optimal path for \((k(t), c(t))\). Now, we will show that this condition will also satisfy the more general transversality condition of (18) i.e
\[
\lim_{t \to \infty} \left[ k(t) \exp \left( - \int_{0}^{t} (r(s) - g) ds \right) \right] \geq 0,
\]
which assumes that households does not asymptotically acquire negative wealth. Using the result of equation (27) that \( \dot{\lambda} = -(r(t) - \rho) \), we can solve for \( \lambda(t) \) to get
\[
\lambda(t) = \lambda(0) \exp \left( - \int_{0}^{t} (r(s) - \rho) ds \right) = U_c(0) \exp \left( - \int_{0}^{t} (r(s) - \rho) ds \right)
\]
Plugging this in equation (20) i.e
\[
\lim_{t \to \infty} \left[ \exp(-\rho g t) \lambda(t) k(t) \right] = 0,
\]
the transversality condition for the optimal path can be written as
\[
\lim_{t \to \infty} \left[ k(t) \exp \left( - \int_{0}^{t} (r(s) - g) ds \right) \right] = 0
\]
This equation is same as the equation (18) without the inequality sign. Thus the stronger version transversality condition can be written as (20), which guarantees unique optimal path \((k(t), c(t))\) for the household optimization problem.

Since \( r(t) = f'(k(t)) - \delta \), transversality condition becomes
\[
\lim_{t \to \infty} \left[ k(t) \exp \left( - \int_{0}^{t} (f'(k(s)) - g - \delta) ds \right) \right] = 0
\]
For the model being considered, we will return to the specifics of transversality condition in the next section when we analyze steady-state thoroughly.

3 Analysis

3.1 Steady State

At steady-state average effective consumption and capital growth rate becomes zero. Thus, from equations (17), (31) and (32)
\[
\dot{k}(t) = f(k(t)) - (g + \delta)k(t) - \tilde{c}(t)
\]
and
\[
\frac{\dot{c}}{c} = \frac{1}{\alpha} \left( r(t) - \rho - \alpha(1 - \theta) \frac{d}{d} \right) = g
\]
Figure 1: Dynamic response of capital and consumption to changes in $\alpha$. Adapted (although used in a different context) from Acemoglu, Introduction to Modern Economic Growth, 2009, Fig. 8.2

Note the distinction between average effective consumption $\tilde{c}(t)$ and average consumption $c(t)$. Whereas average effective consumption $(\frac{c(t)}{A(t)L})$ becomes zero at steady state, average consumption $(\frac{c(t)}{L})$ reaches a constant value of growth rate of technology, given by the equation (32). Using the equation of motion of desired consumption (33) in (43), we get

$$\frac{1}{\theta} (r(t) - \rho - \alpha(1 - \theta)g) = g$$  \hspace{1cm} (44)

Hence the equilibrium market rate of return to capital $r(t)$ can be solved from (44) as

$$r(t) = \rho + \theta g + \alpha(1 - \theta)g$$ \hspace{1cm} (45)

Since market rate of return is given by $r(t) = f'(k(t)) - \delta$ we have,

$$f'(\hat{k}(t)) = \delta + \rho + \theta g + \alpha(1 - \theta)g$$ \hspace{1cm} (46)

Now given the functional form of average effective production function ($f(\hat{k}(t))$), we can solve for the steady state capital from this equation. Similarly from equation (42), the steady state of consumption would be given by

$$\hat{c}(t) = f(\hat{k}) - (\delta + g)\hat{k}$$ \hspace{1cm} (47)

From equation (46), transversality condition (41) becomes,
\[
\lim_{t \to \infty} \left[ k(t) \exp \left( -\int_0^t (\rho - (1 - \theta)g + \alpha(1 - \theta)g)ds \right) \right] = 0
\]  
(48)

This can hold only when the exponent goes to negative infinity. In other words,

\[
\rho - (1 - \theta)g + \alpha(1 - \theta)g > 0
\]  
(49)

This thus implies

\[
\rho > g(1 - \theta)(1 - \alpha)
\]  
(50)

Following the spirit of neoclassical and solow growth model, we now sketch out plot for optimal consumption \( \hat{c} \) and \( \hat{k} \) in the phase space \((k(t), \hat{c}(t))\) using equation (42) and (43). The plot is given by the figure 1. The inverted U-shaped curve is the locus of points when \( \dot{k} = 0 \), given by equation (47). The vertical line is plotted from the condition where \( \frac{d\hat{c}}{dt} = 0 \) provided by the equation (46), which only depends on \( \hat{k} \) at steady state. Mathematically from the equation (46),

\[
\hat{k} = f^{-1} \left[ \delta + \rho + \theta g + \alpha(1 - \theta)g \right]
\]  
(51)

Hence the intersection of these two lines gives us the steady state combination of effective per capita consumption and capital \((\hat{k}, \hat{c})\). The slanted line with the arrow sign represents the locus of points given by the tranversality condition.

### 3.2 Role of the social factor \([\alpha]\)

Now we can do the qualitative study of how this steady state \((\hat{k}, \hat{c})\) varies when parameter \( \alpha \) changes. Recall that \( \alpha \) is the weight given to the comparison utility. In other words, higher value of \( \alpha \) implies that the household values relative consumption between actual consumption and perceived ideal consumption more strongly. As we discussed in Section 1 that different places and culture weight weight the relative consumption differently, we can study cross-country and cross-cultural growth dynamics through \( \alpha \).

From equation (46), we know that

\[
f'(\hat{k}) = \delta + \rho + \theta g + \alpha(1 - \theta)g
\]  
(52)

Taking the derivative of (52) with \( \alpha \) gives us

\[
\frac{f'(\hat{k})}{d(\alpha)} = (1 - \theta)g,
\]  
(53)

For positive growth rate \( g :\) \( \frac{f'(\hat{k}(t))}{d(\alpha)} \) is positive, as \( \theta \in (0, 1) \). This means that increase in \( \alpha \) increases \( f'(\hat{k}) \); but since \( f(k) \) is concave, increasing \( \alpha \) will decrease \( \hat{k} \). In addition to this, lower value for \( \hat{k} \) means lower steady-state consumption \( \hat{c} \) from equation (47). Figuratively this implies that as \( \alpha \) gets higher, it shifts the vertical line to the left which causes the economy to find in lower steady-steady point \((\hat{k}, \hat{c})\). As a matter
fact, when $\alpha = 0$, the steady-steady point ($\hat{k}, \hat{c}$) is maximum for all $\alpha$ and matches the neoclassical growth model that does not consist of comparison utility. In similar fashion, lowering $\alpha$ implies that the vertical line shifts to the right, as a result of which the economy find itself in a higher steady-state ($\hat{k}, \hat{c}$). This is shown in figure 1.

One way to interpret this finding is that $\alpha$ interacts with $\theta$, which in neoclassical growth model is interpreted as household’s intertemporal elasticity of substitution in consumption – i.e household becomes risk-averse to unwilling to substitute consumption over time when $\theta$ increases. We verify this by rewriting equation (50) as

$$\theta > 1 - \frac{\rho}{g(1-\alpha)}$$  

(54)

So $\theta$ increases with $\alpha$. In other words, having higher degree of importance to relative consumption implies that households are more unwilling to substitute consumption over time. His interpretation for $\alpha$ is not unfounded. Constantinides (1990) uses similar comparison utility and interpret $\alpha$ similarly to analyze the effects of habit formation to explain the equity premium puzzle. However, it should be noted that this interpretation should not be thought of as if $\alpha$ and $\theta$ are same. Although higher $\alpha$ implies higher $\theta$ given that everything else remains the same, it does not mean $\alpha$ is the same as $\theta$ – causality does not imply equivalency. What it shows is that individuals willingness to substitute consumption intertemporally i.e $\theta$ is highly underestimated in contemporary growth theories that include $\theta$.

However, more accurate interpretation for $\alpha$ would be to think of it as some measure of household’s degree of impatience. Since we know that higher $\alpha$ means that households are care more about their consumption relative to their ideal consumption, higher $\alpha$ can be interpreted as if households are desperate to consume at their ideal consumption level. Thus households becomes more impatient with higher $\alpha$. Hence higher impatience can make individuals more unwilling to substitute its consumption over time. Given this it means that we have one more variable which incorporates individual’s outlook in growth theory. We can thus appreciate the importance of how individual’s outlook in growth dynamics.

Now given that $\alpha$ can be interpreted as household’s impatient level or degree of risk-averseness of their consumption over time, households with higher $\alpha$ tend to consume impatiently. Thus households tend to save less, as a result of which the economy find itself in a low equilibrium. This can be confirmed by observing the equation for market return to capital $\hat{r}$ (45) in the steady-state. We see that higher $\alpha$ implies higher rate of return. But, since the market is competitive and households supplies the capital to firms, it can be interpreted as if capital are short-supplied as a result of which market rate of return to capital increases.

Now consider a case where $\alpha$ is maximum and is exactly 1. This is where households only cares about their relative consumptions. At steady state level both actual and ideal consumption grow at the same rate (see equation 34). This implies that higher consumption does not provide higher utility to households with $\alpha = 1$, as consumption and ideal consumption are both growing at the same rate. If we were to think of utility as some measure of well-being, this implies that households well-being does not increase with growth, which is commonly known in economics literature as Easterlin’s paradox. So if Easterlin’s paradox was completely accurate, one might interpret through this model that Easterlin’s households only care for relative consumption and ignores their actual consumption. This view is supported by Brickman and Campbell (1971) or equivalently by the term “Hedonic Treadmill”. Unfortunately, if Easterlin’s paradox
is correct, it can be concluded from the model that the steady state level of the economy is in the lowest possible equilibrium point since $\alpha$ is maximum.

Similarly, lower weight in relative consumption $\alpha$ can be interpreted as households with high degree of willingness to substitute their consumption intertemporally or with high degree for patience. Thus households save more, as as a result economy is in relatively higher steady state equilibrium. This result is also confirmed from Solow growth model where saving rate plays a crucial role in economic growth. What remains to be done is to graphically analyze the equilibrium steady level with all possible values for $\alpha$ and $\theta$. This we will do in the next section.

But now, one might wonder if it is possible to shift the vertical line in Figure 1 such that the economy steadily consumes at $k_{gold}$, where consumption $\bar{c}$ is maximum. But, transversality condition (50) restricts the economy from consuming at $\bar{c}$ at $k_{gold}$. We show this by noting that from equations (42) and (52), the following has to satisfy for $(k_{gold}, \bar{c})$

$$\frac{d(\bar{c})}{d(k)} = f'(k) - (g + \delta) = 0$$

or

$$f'(k_{gold}) = (g + \delta)$$

and

$$f'(\bar{k}(t)) = \delta + \rho + \theta g + \alpha(1 - \theta)g$$

At $(k_{gold}, \bar{c})$ these equations (56) and (57) are equal (zero). Hence at $(k_{gold}, \bar{c})$,

$$\alpha = 1 - \frac{\rho}{(1 - \theta)g}$$

But equality sign for this equation does not hold as a consequence of transversality condition (50) i.e

$$\alpha > 1 - \frac{\rho}{(1 - \theta)g}$$

Thus steady-state consumption at $k_{gold}$ is not possible. Note that since this is true for both signs of $g$ and all possible values for $\theta$, there is no optimal combination of $\alpha, g$ and $\theta$ for which the economy can steadily consume at $k_{gold}$.

**For negative growth rate $g$:** For a negative growth rate, the qualitative analysis reverses as the sign of the derivative $\frac{d(k(t))}{d(\alpha)}$ in equation (53) changes. So when the economy is negatively growing, higher alpha will lead to higher steady-state level for $(k, \bar{c})$.

Interestingly we can conclude that Easterlin’s economy, where $\alpha = 1$ or maximum, finds itself in the highest equilibrium point possible when growth is negative. What we can tell from this qualitative observation is that when the economy is growing, it is best to ignore relative consumption, whereas one should care only with relative consumption when the economy is depleting. Philosophical implications of this observation will be left to the readers.
3.3 Parametric Analysis and Simulation

We now look for parametric solution to the model. In order to do so, we first specify the specific function for production function. For Cobb-Douglos production function,

\[ F(K, AL) = K^\gamma (AL)^{1-\gamma} \]  

we can write this in terms of effective average production

\[ y(k(t)) = k^\gamma, \]

we solve for equation (46) and (47) to get

\[ \hat{k} = \left( \frac{\gamma}{\delta + \rho + \theta g + \alpha (1-\theta)g} \right)^{1/\gamma} \]  \hspace{1cm} (62)

and

\[ \hat{c} = \hat{k}^{\gamma} \left( \frac{\gamma}{\delta + \rho + \theta g + \alpha (1-\theta)g} \right)^{1/\gamma} \]  \hspace{1cm} (63)

We now draw three dimensional graphs and contour plot of \( \hat{k} \) and \( \hat{c} \) for all possible values of \( \alpha \) and \( \theta \) and specific values for other parameters in the model.

**Positive growth rate g :-** We first analyze the model for positive growth rate. Parameter values are given by Table 1

Table 1: Parametrization

<table>
<thead>
<tr>
<th>g</th>
<th>δ</th>
<th>ρ</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>.02</td>
<td>.03</td>
<td>.03</td>
<td>.5</td>
</tr>
</tbody>
</table>
Here Figure 2.1 represents a 3D plot for the model’s equilibrium capital $\hat{k}(\theta, \alpha)$ and values of parameters given by Table 1. Figure 2.2 refers to the contour plot of Figure 2.1, where light area indicates high $\hat{k}$ and where $\alpha$ is represented by $y$-axis and $\theta$ is along the $x$-axis. So, we confirm our earlier qualitative observation that with positive growth and everything else constant, higher $\alpha$ leads to lower level of equilibrium capital $k(\theta, \alpha)$. As a matter of fact, $k(\theta, \alpha)$ is maximum when both $\theta$ and $\alpha$ is near minimum. Similar observation is made when we look at Fig 3.1, which is a contour plot of $\hat{c}(\theta, \alpha)$ for Table 1. Figure 3.2 is the contour plot of the ratio of household equilibrium consumption to capital $\frac{\hat{c}}{\hat{k}}$. Figure 3.2 shows that household does consume more when $\alpha$ and $\theta$ are higher as the it ratio $\frac{\hat{c}}{\hat{k}}$ is higher when one moves away from the origin. So we can conclude from this observation that during positive economic growth, the economy is best served when households are willing to substitute consumption over time and do not care about their relative consumption. See above for Section 3.2 Role of Social factor for its qualitative explanation.
Negative growth rate $g$ :- Now for the economy with negative growth however, the analysis and conclusion is reversed as predicted from the previous section. Figures 4 and 5 follows in similar fashion as the analysis for positive growth expect that the capital $\hat{k}$ and $\hat{c}$ are highest when $\alpha$ and $\theta$ are maximum. So we can conclude from this observation that that during negative economic growth, the economy is best served when households are highly risk averse to substitute consumption over time and care deeply about their relative consumption.

Table 2: Parametrization

<table>
<thead>
<tr>
<th>$g$</th>
<th>$\delta$</th>
<th>$\rho$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.02</td>
<td>.03</td>
<td>.03</td>
<td>.5</td>
</tr>
</tbody>
</table>
Figure 4.1: $k(\theta, \alpha)$ for Table 2

Figure 4.2: Contour Plot $k(\theta, \alpha)$ for Table 2

Figure 5.1: Contour Plot $\hat{c}(\theta, \alpha)$ for Table 2

Figure 5.2: Contour Plot $\frac{1}{2}$ for Table 2
4 Conclusion

In this paper, what we have done is create a model to understand how material norms and aspirations on which individual judgments are based plays a role in the growth dynamics. Through our model, we explained how an economy that values strongly its relative consumptions, where the reference is given by the perceived ideal consumptions is worse off than an economy that values relative consumption less strongly. This is true when the economy is growing at a positive rate. During negative growth however, we showed that it is best for individuals in the economy to care only with its relative consumption. Combining these two observations, we can conclude that when the economy is growing, it is best to ignore relative consumption, whereas one should care only with relative consumption when the economy is depleting.

This paper also shows that individual’s outlook and social norms plays an important role in economic growth than we have previously thought of. We started out with a goal to explain cross-cultural and cross-country growth dynamics, provided that the perception of relative consumption varies across places and social norms. Incorporating comparison utility into a growth model provided us with a rich sets of tools and techniques to study this cross-cultural and cross-country growth mechanics. Thus, this model serves as an excellent framework for studying cross-cultural growth. Although we showed theoretically that individual perception of material norms and aspiration plays a significant role in economic growth by incorporating social factors in a growth model, what remains to be done is an empirical study to look how our conclusions converge with the empirical results.
References


