1. TECHNICAL INFORMATION

Instructor: Josh Sabloff (jsabloff@haverford.edu)
Office Hours: M 1-3:30, Tu 2-4, W 12:30-2 in KINSC H213
You may also want to consult:
(1) do Carmo, Differential Geometry of Curves and Surfaces. Prentice Hall, 1976. I believe that there is now a new Dover edition of this book as well.
(2) Shifrin, Differential Geometry: A First Course in Curves and Surfaces. Notes available at:

2. GOALS OF THE COURSE

2.1. Meta-Goals. As with every course in the Math Department, the primary goal of this course is to help you learn to think like a mathematician. Since Math 337 is a 300-level elective, I will assume that you are already comfortable with the language of definition, theorem, and proof from your earlier classes (though I expect that not a few of you will want to polish your skills to one extent or another). In essence, you have mainly used this language as a method of analysis, i.e. breaking an idea into its component pieces and thinking about the connections between them. In this course, I hope you will begin to deepen your mathematical thinking by bringing larger concepts together, be they from prior courses (multivariable calculus, linear algebra, and analysis) or from within the field of differential geometry. You will also work on picking a pathway through a mass of mathematical material.

To be clear, we are by no means abandoning the usual rigorous mathematical thinking you have grown to love, as the mathematical endeavor is founded on rigorous proof!

As always, I hope to help you improve your problem-solving techniques, your written and oral mathematical communication skills, and your ability to visualize (and/or your ability to appropriately farm out visualization to Mathematica). Note that differential geometry is a relatively computational and example-based field by tradition and by nature, so do not be surprised if some parts of the course remind you of your 121/216 days!

2.2. Mathematical Content. There are two sets of contrasting perspectives in differential geometry: local vs. global and intrinsic vs. extrinsic.

Local, or descriptive, geometry puts forth language to describe properties the geometry of a curve or (more interestingly) a surface in a neighborhood of a point.
There are two ways to develop this language: extrinsically (using the way the surface is embedded inside of $\mathbb{R}^3$) or intrinsically (using only quantities that can be measured on the surface itself). We will take both approaches, and show that they are equivalent approaches to the definition of curvature — this is Gauss’ famous *Theorema Egregium*.

Global geometry asks how local properties influence the behavior of the entire surface. We will see a few examples of this to whet your appetite, including the Fary-Milnor Theorem about the total curvature of knotted curves and the Gauss-Bonnet Theorem, the Coolest Theorem in Undergraduate Mathematics.™ What does it say? You’ll find out!

2.3. **Prerequisites.** As stated in the course catalog, you should have taken Math 317 (or, with my consent, only Math 216). In particular, I assume that you are comfortable with the mechanics of multivariable calculus (though I will do some quick review as needed so that we can agree on notation), linear algebra (especially inner products and quadratic forms), and the basic topology of $\mathbb{R}^n$ (open, closed, compact, connected) as laid out in Math 317.

3. **Course Structure**

3.1. **“Daily” Assignments.** For each class meeting, you will be expected prepare as follows:

   (1) Read the relevant pages of the text(s) and briefly (!) answer the standard questions on Moodle by 11:55p the night before class:

   (a) What was the most important point (definition, theorem, etc.) in today’s reading?
   (b) How does today’s reading connect to previously discussed material?
   (c) What was the most confusing point? What questions do you have?

   (2) Write up solutions to the posted exercises. There will generally by 2–3 exercises: one or two “preview” exercises about the day’s reading and one or two “review” exercises designed to solidify the previous day’s discussion. I reserve the right to re-assign a preview exercise as a review exercise. You should hand in your solutions on Moodle by 11:30a on the day of class.

   The reading questions and preview questions will be graded on a “complete / incomplete” scale (where “complete” means that you gave the questions a serious effort, but you need not get them correct), while the review questions will be graded on a 0–4 point scale.

3.2. **Mini-Projects.** To complement the daily assignments, you will complete a mini-project every three weeks that allows you to delve more deeply into an interesting class of examples or a piece of theory that we did not have time to cover in class. I will give you several options each time, but only a certain number of people can work on each option. Email me with an ordered list of your choices; first come, first served.
3.2.1. Written Report. Your written report should completely and clearly answer the question proposed, give context for the material, and be aimed at an audience of your peers. Some (though not all) of the mini-projects will require you to do some research in other texts or in the primary literature.

The mini-project reports should be typed in \LaTeX\ and submitted electronically on Moodle. Reports should be about 2–4 pages in length. I will provide sample and template files for you to use. If you have never used \LaTeX\ before, please see:

http://www.haverford.edu/mathematics/resources/LaTeX.php

and/or talk to me.

The paper should be clearly and carefully written and adhere to professional mathematical standards for written mathematics. In particular, you must engage with precise definitions of terms, make careful statements of theorems, and write correct and motivated proofs.

3.2.2. Oral Presentation. For one of your mini-projects, you will give a 10-minute oral presentation of your report. The time constraint means that you cannot say everything, and certainly cannot give all details! What is the most important idea? What techniques did you use? How does your mini-project relate to other material from class?

You are more than welcome to discuss your presentation with me beforehand.

3.3. Exams. There will be a take-home, open book final exam. There will be no other exams in this class.

3.4. Late Work Policy. The best way to do well in this course is to keep up with the daily assignments and mini-projects (even if you do not finish every problem). Nevertheless, your schedule can sometimes get crazy, so I will accept up to two late daily assignments up to the next class meeting after the original due date without penalty if you tell me beforehand.\footnote{It is not necessary to use this extension when dealing with a serious illness — sniffles don’t count — a family emergency, or a religious holiday.} Other late work will be accepted for a 50% penalty.

3.5. Class Time. Class time will be a mix of traditional lecture and small-group discussion. We will begin with a discussion of the day’s topics based on your responses; I hope to gradually cede more control of this portion of the class to you over the course of the semester. We will then transition to a bit of lecture whose content will vary (perhaps an explanation of a difficult proof, perhaps an interesting example, or perhaps a connection to further or related topics). Finally, we will move to small-group discussions of examples and propositions to clarify concepts presented in the brief lectures. Some days, we might cycle through more than one round of lecture and discussion.
4. Collaboration Guidelines

Collaboration with your classmates on the homework assignments — whether in the MQC, in office hours, or on your own — is allowed, even encouraged. There is a fine balance, however, between learning from working with your fellow students and finding your own way through the material, and I encourage you to keep this in mind and reserve significant "alone time". You must indicate on your homework who your collaborators were (you may note different collaborators on different problems!). You should follow these guidelines in your collaborations and write-ups (instructions in italics are not strictly required, but can be helpful in following the guidelines):

- You must work on the problems on your own before talking with your collaborators. Use white paper.
- Bring your ideas to your collaborators. If you don’t have any ideas to bring for a problem — though note that even knowing where to start, what you need to do to solve the problem, or having a vague idea about how the solution should go is an “idea” — then you should wait until you do before collaborating on that particular problem. Use colored paper, a chalk board, or a white board; the same applies when you discuss problems with me.
- The actual write-up of your homework assignment should be done with all colored paper put away and out of sight of any black/whiteboards used during collaboration so as to reflect your own understanding of the problem. If you cannot write up the solution without wanting to refer to your collaboration material, then you probably have not yet understood the problem. In that case, throw out your solution, work further on your own, and then start the collaboration process again.

Note that this means that while you are collaborating, you cannot be working on the final draft of your problem set!

Collaboration on certain parts of the mini-projects is allowed as well. You may collaborate during the process of understanding the material (working out definitions, working through examples, working through proofs), and you may have a classmate read and comment upon a complete draft, but your write-up must be completely your own. That is, you must be working alone from the second you set finger to keyboard.

5. Grading

Your grade for the course is derived from the following weighting:

**Daily Reading Assignments:** 10%
**Daily Homework Assignments:** 30%
**Written Mini-Projects:** 40% (the first will only count as 10% to give you room to figure out the format; the next two count as 15% each)
**Oral Presentation(s):** 5%
**Final:** 15%
6. Resources

Haverford College is committed to supporting the learning process for all students. Please contact me as soon as possible if you are having difficulties in the course. There are also many resources on campus available to you as a student, including the Office of Academic Resources (https://www.haverford.edu/oar/) and the Office of Access and Disability Services (https://www.haverford.edu/access-and-disability-services/). If you think you may need accommodations because of a disability, you should contact Access and Disability Services at hc-ads@haverford.edu. If you have already been approved to receive academic accommodations and would like to request accommodations in this course because of a disability, please meet with me privately at the beginning of the semester (ideally within the first two weeks) with your verification letter.