Math Education in the U.S.: A Discussion of History, Principles, and Practice

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Thesis for the Special Major in Educational Studies and Mathematics

With Advisors Ann Renninger and Ralph Gomez

Swarthmore College, Spring 2017

Acknowledgments:

I would like to thank my advisors, Ann Renninger and Ralph Gomez, for their invaluable support and suggestions as I continued to work on this thesis. I'd also like to thank the entire Educational Studies department, professors and students alike, for inspiring me to pursue this field, as well as for energizing me with copious amounts of food and tea. Finally, I'd like to thank Professor Emeritus Charles Grinstead, for telling me in my first semester for giving me the outrageous idea that if I wanted to be a math major, I could.

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Introduction

Many U.S. citizens, when asked to name their least favorite subject in school, will answer "Math". In the U.S., secondary mathematics is perceived as boring, difficult, procedural, and entirely unrelated to the real world. Weiss et al's (2003) study also found 59% of math and science classrooms to be "low in quality", describing the lessons as unlikely to enhance student understanding. They also found that fewer than 1 in 5 lessons observed in the study were "strong in intellectual rigor", included useful teacher questioning[1], or provided appropriate sense-making. Given this, it is unsurprising that so many U.S. students are struggling with mathematics; only about a quarter of twelfth graders are considered Proficient by the National Assessment of Educational Progress (NAEP[2]), and White students outperform Black and Latino students by a wide margin (Bohrnstedt et al, 2015). Qualitative data echoes the reality that students taught in traditional classrooms often dislike math and are unable to apply it outside of the classroom (Boaler & Selling, 2017). This is particularly unfortunate in the 21st century, when increasing dependence on technology is causing jobs openings in STEM to grow twice as fast as jobs in other sectors, as well as to pay up to 26% more (U.S. Department of Commerce, 2011).

Of course, success in math begins in the classroom, with students developing knowledge, skills, and mindsets around math from the time they enter school until after they leave it. It is therefore the responsibility of schools and teachers of mathematics to make math education both

[1] Where teacher questioning is not defined as procedural questions, such as "What is 2 and 3?" but questions with the purpose of enhancing student understanding, such as "What representation might we want to use to help U.S. solve this problem, and why?" (Leinwand et al, 2014)

[2] The NAEP was first developed during the push for math reform in the sixties, but was not regularly given to students until the 1990s, after which U.S. students in grades 4, 8, and 12 were assessed every two to four years.

more engaging and more equitable. This leads me to ask the question: What teaching practices can math teachers employ that increase student achievement *and* engagement in math? And furthermore, which of these practices can also help to decrease achievement gaps between students of different racial and socioeconomic groups? I plan to answer these questions not only through theory but also through research, which I will use to provide evidence for the pedagogies that I believe teachers should consider when trying to create challenging, engaging, and equitable math classrooms.

This thesis addresses the aforementioned questions in three parts. The first summarizes the history of math education in the U.S. since the 1950s, discussing past ideologies around the teaching of mathematics and the research, historical events, statistical data, and other factors that fueled them, particularly the influence of the National Council of Teachers of Mathematics[3]. The second section considers math education in the 21st century, including an analysis of current test scores and a discussion of the Common Core State Standards for Mathematics[4] (2010), which I argue that, despite implementation flaws, provides a comprehensive description of what we want students to be able to do with mathematics. Finally, I overview recent classroom studies reporting on teaching practices that increase student achievement, decrease achievement gaps between racial groups, increase student enjoyment and engagement in mathematics, and describe the mindsets that we want to encourage and how these can be fostered.

[3] The world's largest mathematical education organization, NCTM was founded in 1920 and has since published several major sets of math curriculum standards, including the 1989 *Curriculum and Evaluation Standards* (*Standards*), the 2000 *Principles and Standards,* and the 2010 *Common Core State Standards for Mathematics* (CCSSM)

[4] Published by the National Council for Teachers of Mathematics in 2010, the *Common Core State Standards for Mathematics* outline 8 Standards for Mathematical Practice that teachers should be developing in their students, as well as content standards for each grade. As of 2017, forty-two U.S. states have adopted the Common Core. (National Governor's Association Center)

Math Education in the U.S. – A Brief History

Introduction: Recurring Themes

The history of mathematics education has been characterized by frequent changes in focus, with policy vacillating between considering what is best for individual students and considering what is best for the U.S. as a whole, particularly the nation's ability to compete in an increasingly technology- and science-oriented world. Because mathematics is so connected to the nation's technological development, and because technology is always growing and changing, beliefs concerning the importance of mathematics and what students need to be able to do with mathematics are constantly changing as well. Conflicting beliefs between parents, policymakers, government, and students have resulted in reactionary movements, with math pedagogy and content swinging sharply from one extreme to the other. This conflict often manifests itself in complications with creating appropriate assessments; for instance, if someone who views math as a set of procedures creates a standardized test, and gives it at a school where the teachers views math as a set of relationships and concepts, the students may do poorly. But if given a test that assess student understanding of relationships and concepts, the students may do well. This discrepancy makes it difficult to gauge what "high achievement" means, as it requires some agreement on what math is for and what knowledge and skills students ought to have to be "good" at mathematics.

Another recurring theme in the history of mathematics education is the frequent existence of divides between what policymakers believe should take place in the classroom and what teachers actually know how to do. This could be a gap between the content that teachers must teach and what content they actually know, or between what pedagogies they are *supposed* to use and the pedagogies they actually know *how* to use. It has often been the case that policies have failed or at least struggled to be properly implemented because there wasn't enough professional development to help those teachers who needed to incorporate the new policies into their classrooms (Woodward, 2004). Furthermore, as in the case of CCSSM, there has been difficulty creating textbooks and other instructional materials that align with the new policies in ways that the writers believe to be appropriate.

Many of these themes—conflicting beliefs about math education, discrepancies between assessments, and divides between policy and implementation—occur again and again in math education's history. It is important to consider these problems, as it seems likely that they will continue to occur whenever a new policy goes into place. While optimally, the latest research in math education pedagogy should be enough to help our students to succeed, the reality is that many different factors come into play in the vast space between what policymakers dictate, and what actually takes place in the classroom.

Global Competition: The New Math Era

This is not the first time in America's history when math educators, mathematicians, the federal government, and the American public all felt that there was something wrong with math education. One of the more memorable and impactful instances of math education reform took place roughly from 1957 to 1965, in a diverse series of textbook, curricular, and pedagogical reforms that became cumulatively known as New Math. A movement characterized by its emphasis on abstract mathematical concepts in the early grades, its de-emphasis on procedural knowledge and memorization, and its promotion of Discovery Learning[1] (Woodward, 2004; Bellos, 2014), New Math was spurred into existence both by the growing belief that the rote memorization taking place in classrooms was not producing critical thinkers, and hurried along when the USSR released Sputnik in 1957, convincing the U.S. government that our nation was falling behind and that improving student understanding of mathematics, as well as nurturing more future mathematicians and scientists, would help U.S. to compete in a more technologically oriented and competitive world (Woodward, 2004; Bellos, 2014, Bossé, 1995, Philips, 2015). As a result, the U.S. government provided federal funding to produce more math and teacher educators; this allowed the NSF to found the School Mathematics Study Group (SMSG), a group of professional mathematicians headed by Edward G. Begle who shared the goal of reforming K-12 education (Loveless, 2001). This group focused on developing new curriculum for K-12 students that echoed what students need to know to pursue math and science at the college level. In teacher training, the group encouraged new mathematical practices, including Bruner's Discovery Learning, that they felt would both enhance student understanding and increase student's abilities to use mathematics outside the classroom. Teachers were also encouraged to focus on instruction in "structure, proof, generalization, and abstraction" (Woodward, 2004),

[1] Introduced by Jerome Bruner in the 1960s, Discovery Learning is an inquiry-based theory of instruction which argues that rather than being receptacles for information, students should draw on their past experiences and current knowledge to discover new knowledge, through asking questions, forming hypothesis, and solving problems. The theory was backed by current research in learning psychology, and in terms of mathematics, pushed against rote learning, procedural knowledge, and memorization as classroom practices. (Woodward, 2004; Bruner, 1960)

concepts that many mathematicians would agree are central to doing mathematics but were presently under-emphasized in traditional K-12 classrooms.

Research that supported the development of these curricula included that completed by current educational psychologists, who encouraged a move away from behaviorist practices and supported psychologists who were working to develop pedagogies that taught students the structure of subjects, rather than discrete facts. Bruner drew on his own and other psychologists' research into the notion of transfer, and the ways in which we can help students transfer concepts that are core to a discipline from what topic to another (Bruner, 1960). This research formed the foundation for the way in which math was *meant to be* taught in the New Math era, though as mentioned previously, implementation complications suggest that not all teachers actually took up these new pedagogies successfully.

Other studies were conducted with a focus on content, such as the research carried out by the 1959 Commission on Mathematics of the College Entrance Examination Board (CEEB, 1959; Fey, 1978), which studied college entrance exams in order to determine what topics students should focus on in secondary school. SMSG went on to develop course materials that would support CEEB's findings. But their focus was much more on what students should be learning to prepare them for college (like Modern Algebra, statistics, and probability), rather than how it should be taught, and teacher training (where it existed) had a similar emphasis. Teachers were trained more to understand the new subject material than in the ways in which they should present the material. Because of this, the New Math movement experienced the barrier, remarkably similar to that of Common Core, that teachers were not prepared to teach the curriculum as effectively as the mathematicians who developed it would have liked. The movement required that teachers reconceptualize their understanding of both mathematics and mathematics teaching. But the costs for professional development were too high and therefore inaccessible to the many thousands of teachers who needed it (Woodward, 2004; Fey, 1978).

Later research attempted to determine what effect the "New Math" curricula had had on student achievement using the new phenomenon of standardized tests. Evidence for New Math's failure consisted of the fact that California state math scores had dropped in the three years since 1968, and that in New York, a third of sixth-graders were underperforming in mathematics (Philips, 2015). However, these results are complicated; in California, state test scores in *all* subjects were declining, and that in New York, there was no evidence to suggest that students who used New Math textbooks were faring worse than those who didn't (Philips, 2015). Furthermore, the tests given were tests of computation, which largely overlooked the fact that New Math was placing more emphasis on conceptual understanding. Supporters of the movement had in fact assumed that computational skills would decrease, and were willing to accept this, because they did not believe computation encompassed the purpose of mathematics. To challenge the earlier finding, evaluators altered the test to better align with what New Math was meant to be teaching (Fey, 1978, p. 350). But the damage was done; once the media spread the "news" that students were faring worse or computational tests as a result of New Math, parents were ready for the movement to end—which it did (Fey, 1978; Philips, 2014).

Due to the complications above, as well as to the fact that the Vietnam War caused SMSG to lose its funding and subsequently its ability to continue professional development, New Math did not continue into the 70s (Philips, 2014). Still, the New Math movement succeeded in one way—it drove mathematics, at least temporarily, away from a focus on arithmetic and drill, and toward a focus on thinking and reasoning. Math educators began to think more critically about what the link should be between the math taught in classrooms and the math that working mathematicians must do. Unfortunately, one of the reasons that U.S. math classrooms continue to be focused on memorization and procedure is due to the movement on the heels of New Math, known as Back to Basics, which I will describe in the section that follows.

Back to Basics: A Reaction

As discussed above, many states reported that scores on computational tests fell during the time of the New Math movement, and argued that this was a sign of the movement's failure. In an effort to compensate, many people began to believe that schools ought to focus on "the basics", in terms of reading, writing, and math. Furthermore, it became clear to education reformers that in inner-city schools, many students were failing to achieve even basic levels of mathematical knowledge (Woodward, 2004; Philips, 2014). Whereas the 60s were about competing internationally, the 70s became focused on reform at home. Although there wasn't a unified movement as with New Math, the policies and attitudes of separate school communities began to swing in the same direction, focusing on memorization, drill, behavioral improvements, and the "three r's" (reading, 'riting, and 'rithmetic). Programs in art, sex education, and dance were cut, and teachers congratulated themselves on their "traditional" teaching methods. The movement was also focused on accountability—only standardized tests, oftentimes tests focused on arithmetic, could tell U.S. whether or not low-performing schools were seeing improvement (Woodward, 2004).

The conflict between New Math and Back to Basics, between focus on simple computation and conceptual understanding of high-level math, is perhaps epitomized by the experience of a chemistry professor named James M. Shackelford, reported by Philips (2015).

Shackelford, upon examining his 4th-grade daughter's mathematics textbook in 1972 discovered that there were questions about set theory[2] that he could not understand, and few questions focusing on memorization. Shortly he discovered that neither his daughter nor her peers were able to calculate 8x9. It was this kind of disparity that ushered in the Back to Basics movement; parents and teachers cared less about students having a structural understanding of mathematics, and more about their ability to execute fundamental skills and memorize times tables. In his critique, Shackelford made the point that "the most abstract math he used was that needed to figure out the grocery bill"; to him, the ideas emphasized in the New Math movement were largely irrelevant. Just like the supporters of the New Math movement, those who supported a "Back to Basics" approach were just trying to make sure that students possessed the mathematical abilities they believed to be important. The problem was that the movements had extremely different definitions for what those were; New Math supported abstract knowledge and reasoning, while Back to Basics supported memorization and calculation.

Another difference between New Math and Back to Basics is the recommended pedagogy; whereas the pedagogy of the New Math era was fueled by the research of Bruner, the pedagogy of Back to Basics was fueled by the research of behaviorist psychologist Skinner (1968), who introduced the idea of Operant Conditioning, the use of rewards to support learning. Behaviorist teaching practices did not treat students as people who construct knowledge, as Bruner emphasized; rather, they argued for the provision of information in order to develop knowledge. According to Skinner, students need to be able to replicate necessary procedures, and that they can be trained to do this, even without understanding the reasoning behind them (Skinner, 1968).

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[2] Set theory is the branch of mathematics that studies sets, which are best described as collections of objects

The Back to Basics era possessed a wide foundation of classroom-based studies focused on the use of behaviorist practices, which the government believed could more successfully teach math to students in inner-city schools. These included the 1972 Missouri Mathematics Effectiveness Project, which sought to directly connect teaching practices in the fourth grade with outcomes, i.e., with scores on standardized tests. For the study, researchers observed elementary math teachers at two urban, low-income schools and categorized them as either "effective" or "ineffective"; they then incorporated the practices of the effective teachers into an instructional program, which they introduced to a Treatment group at one of the schools; the teachers at the other school did not learn about the instructional program, and acted as the Control. Practices were largely concerned with the way math class was structured, and included components like the conducting of Daily Review, development of the lesson with "lively" demonstrations, Seatwork, and Homework Assignments; each part of the lesson was supposed to receive a specified amount of class time (Good & Grouws, 1979). The study found that most teachers in the Treatment group implemented the instructional practices, assigning the appropriate amount of time to each component of the lesson, and that the classes that experienced the treatment scored better on a subsequent standardized test called the Science Research Associates Mathematics Achievement test, in addition to a test of content.

Also during this time, an extensive, federally funded research study called Project Follow Through was launched, with the goal of determining the "most effective methods" for teaching basic skills to at-risk elementary school students in economically disadvantaged areas. The study analyzed the effect of several teaching models on students' standardized test performance, including instruction that focused on "basic skills", i.e., the mechanics of spelling, and simple arithmetic; instruction that focused on "cognitive-conceptual" abilities, i.e. reading comprehension and math problem solving, and "affective-cognitive" abilities, i.e., self-concept and attitudes toward learning (House et al, 1978). According to newspaper articles published at the time, the study concluded that Direct Instruction, in which teachers and aids taught scripted lessons to small groups, focusing on basic skills and arithmetical procedures, and using behaviorist practices, was most successful (Woodward, 2004; Meyer et al, 1983).

However, the true findings are more difficult to parse. This is in part because of the way the different models for instruction were defined, implemented, and tested. Student ability was evaluated using the MAT (Metropolitan Achievement Test), which contains basic questions on spelling, computation questions in math, but also math application problems and reading comprehension questions (House, 1978; Meyer et al, 1983). Because not all of these skills were categorized by Project Follow Through as "Basic", the researchers defined "Basic Skills" as being tested by basic spelling and reading questions and computation questions; the others fell in the realm of "cognitive-conceptual". Essentially, the test defined "cognitive-conceptual" skills as those that cannot be directly taught, or that cannot be taught "by rote", i.e., through behaviorist methods (House, 1978). And while the study did find that a Basic Skills approach was most effective in increasing students' Basic Skills, this is hardly surprising given that other models were not similarly focused. The study also found that no model improved "cognitive-conceptual" skills—but even the cognitive conceptual teaching models didn't line up with the evaluation nearly as much as the basic skills model did.

However, much of the study focused on comparing Follow Through to non-Follow Through programs. Even here, the design of the study could potentially have created invalid results, due to the different ways in which the program selected the number of cities and students to be studied. House et al (1978) describe this by pointing out that, because large sample sizes will find significant results for smaller differences, if a study compared 50 groups of 20 students each, and another study compared 10 groups of 100 students each, the latter study is much more likely to find significant results due to the larger sample size—even though both studies involve 1000 students. Project Follow through regularly divided up sites in this way, making it easy to make significant differences seem insignificant, and vice versa.

Essentially, due to their selections of samples, categorizations of different teaching models, and ways in which they evaluated the models, the results of Project Follow Through are uncertain; but because the project was publicized as a wild success, much of the U.S. public became strong supporters of it, and the program continued for a number of years (House, 1978). Unfortunately, it seems that it was the types of methodologies encouraged by Missouri and Project Follow Through that resulted in teaching practices for inner-city children being so different from those presented to upper-class children, as Anyon[3] discovered in her essay on the Hidden Curriculum in schools. Although the U.S. government and public were perhaps well-meaning, their conclusion that low SES students must be taught in more simplistic ways than high SES students has had unfortunate consequences.

Moving Past the Basics: NCTM Standards

Of course, the Back to Basics movement and support of Basic Skills instruction did not continue forever. Even while the movement was in full force in the 80s, cognitive scientists were

[3] Education Researcher Jean Anyon's 1980 article, *Social Class and the Hidden Curriculum of Work,* discussed her observations of four different public schools with students from four different socioeconomic class backgrounds; the poorest students attended schools where most students were working class, and the wealthiest attended schools where most students' had a parent who was an *executive elite,* i.e., a top executive at a multinational corporation. Anyon found that working class students experienced the most behaviorist pedagogy, and were assigned to complete simple arithmetic and worksheets, with a strong emphasis on behavior, whereas executive elite students experienced more engaging, project-based work that required them to think critically. (Anyon, 1980)

beginning to more carefully study student learning and problem solving, including the importance of metacognition to doing mathematics (Schoenfeld, 1987). Richard Skemp was one such cognitive scientist, and criticized the use of behaviorism in the classroom as well as the practice of having students "do" mathematics without actually understanding it (Woodward, 2004). Behaviorism worked in direct opposition to the dominant Information Processing Theory, which argued that the human mind is like a computer; it doesn't just respond to stimuli as Skinner would suggest, but also processes it. This idea further led math researchers to constructivism, an ideology which argues that not only does the brain need to *process* mathematical knowledge, but also that the brain *constructs* mathematical knowledge; knowledge is reinvented by individual learners in unique ways (Bruner, 1960).

These ideas, coupled with Reagan's educational 1983 report *A Nation at Risk*, which harshly criticized both the Back to Basics movement and the quality of American schools, launched the U.S. into another era of education reform. In a summary of concerns over math education in the 70s, as articulated in various NSF reports and conferences, Fey & Graeber wrote on various problems they saw with U.S. math education, including the fact that the public "views mathematics as a set of arithmetic skills", a mindset that the Back to Basics movement was explicitly fostering. Fey and Graeber added suggestions, such as that applications and problem solving should have a more central role in math education, and that technology needed to be considered "in relation to computational skills and the handling of the 'daily bombardment of statistics'" (Herrera & Owens, 2001). In response to these critiques, as well as nation-wide complaints that the U.S. was still behind other nations in terms of math performance and technology, the National Council for the Teachers of Mathematics established the Commission on Standards for School Mathematics in 1986, with the goal of producing a document that could help improve the quality of school mathematics. This document became the 1989 *Curriculum and Evaluation Standards for School Mathematics,* with a "standard" defined as "a statement that can be used to judge the quality of a mathematics curriculum"; i.e., standards explain what is valued in the learning of mathematics. The document includes thirteen to fourteen standards for each grade, and discusses what the standard implies for practice, and concludes with standards for evaluation (NCTM, 1989).

With the *Standards,* NCTM sought to: "Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied to diverse fields" (Center for the Study of Mathematics Curriculum). The standards outlined goals similar to those emphasized in the 2010 Common Core Standards, stating that students should: "Learn to value mathematics, Become confident in their ability to do mathematics, Become mathematical problem solvers, Learn to communicate mathematically, and Learn to reason mathematically" (p. 5) (Center for the Study of Mathematics Curriculum). These goals differed significantly from the tenants of the Back to Basics movement, which emphasized computation much more than mathematical reasoning and communication.

As part of their goals, the *Standards* aimed to change both Content and Pedagogy. In terms of Content, they did not specify all topics to be covered but created guidelines, which including increasing the topics included in secondary math to include discrete math, statistics, and mathematical modeling, stressing connections between mathematical topics and between math and the real world, emphasizing higher-order thinking and problem solving, and encouraging math teachers in the elementary grades to expand their focus beyond arithmetic to include geometry, patterns, and statistics (Herrera & Owens, 2001).

In terms of pedagogy, the *Standards* urged active student involvement in discovering mathematical relationships and concepts, rather than just memorizing procedures; using graphic and alternative representations to help increase student understanding; encouraging group work; having students write to reflect and communicate on mathematical ideas; the use of context to ground problems; and for teachers to see themselves as facilitators of classrooms, rather than instructors (Herrera & Owens, 2001). Of course, as with New Math, the reforms faced considerable backlash, and gratuitous misinterpretation, with many feeling that there were too many applications and not enough math, or arithmetic, being taught. The *Standards* also lacked specificity, not clarifying what should be taught in which year. In response to these criticisms, NCTM sought to clarify their message with the 2000 Standards, which listed five Content Standards (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) and five Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representations) (NCTM, 2000).

Answering the Research Questions:

Returning to the content of my introduction, I propose my research questions once more: In the history of math education, what teaching practices did math teachers employ to increase student achievement and engagement in math? And furthermore, which of these practices did they also hope would help to decrease achievement gaps between students of different racial and socioeconomic groups? Over the course of the second half of the 21st century, answers to these questions according to educators and policymakers have changed significantly. Proponents of the New Math movement might argue that Discovery Learning could increase both student achievement and engagement, through its alignment with cognitive psychologists' research on the type of classrooms that help student learns most effectively and that also spark their interest. New Math supporters also seemed to believe that a key component of increasing achievement was changing the mathematics taught in schools so that it was of a higher level and required more critical thought and reasoning abilities. The New Math Movement did not appear to concern itself with decreasing achievement gaps; rather, research suggests that many students whose learning styles didn't align with the teaching practices encouraged by New Math fell behind (Woodward, 2004).

Following this, proponents of Back to Basics argued that teachers can best increase student achievement through basic skills instruction, behaviorist teaching practices, and the Direct Instruction Model. They found that carefully structured lessons, and oftentimes scripted ones, which teach students procedures and provide them opportunities to practice through classwork and homework, student test scores increase. The movement didn't seem concerned with student engagement, and no studies that I reviewed included student perspectives on their enjoyment of the class or interest in the material. The Back to Basics Movement was specifically meant to help close achievement gaps, with studies focusing on finding methods that worked for low-income, urban schools. As a result, studies sought to decrease achievement gaps by introducing new pedagogies and interventions only to classrooms that were struggling.

Finally, NCTM's 1989 and 2000 Standards pushed against Back to Basics, arguing that we can best increase student achievement by emphasizing the Process Standards, including Problem Solving, Reasoning and Proof, Communication, Connections, and Representations. Rather than focusing on decreasing achievement gaps, the *Standards* focused on determining what *all* students should know and be able to do as a result of their math education (Crosswhite, et al 1989). Among its list of "New Societal Goals", it included "Opportunity for all", which asserted that it can no longer be the case that most who study advanced mathematics are white males, and that it is now a necessity for women and minorities to “enjoy equal opportunities” through mathematic literacy (NCTM, 1989).

Math Education Today

21st Century Accountability: NAEP and TIMSS

Despite all of these reforms, assessments such as the Trends in International Mathematics Study (TIMSS[1]) and the National Assessment of Educational Progress (NAEP) would suggest that the U.S. is still behind its own goals when it comes to mathematics. Unlike standardized tests in the Back to Basics era, the NAEP as it currently stands doesn't test only computation and arithmetic, but students' ability to apply concepts to solve problems, an ability not emphasized in the Back to Basics movement. The description of students at the Proficient level for each grade emphasize that students should be able to apply concepts and integrate procedural knowledge to complex problems, and in Grade 12, that students can use deductive reasoning. Parts of the NAEP contributed to the 1995 TIMSS, which the U.S. chose to participate in so that we could identify our position in comparison to other nations.

Schmidt's (2012) analysis of the NAEP shows that we have indeed made progress since the 1996 test was given, but not as much progress as we would perhaps like. While the number of fourth graders scoring Proficient has increased from 19% to 33% since 1996, and from 20% to 26% in the eighth grade, these data still mean that only ¼ of students are considered Proficient in math when they enter high school (Schmidt, 2012). This would explain the equally low score among 12th graders, whose Proficiency has only increased from 21% to 23% in 2009. In 2015,

[1] The TIMSS provides data comparing U.S. students' achievement in math compared to that of other nations, and data has been collected from students in grades 4 and 8 every four years since 1995. The most recent collection was in 2015 and included students in grades 4, 8, and 12; more than 60 countries participated (https://nces.ed.gov/timss/)

this number only rose to 25%—not even a significant difference (NCES, 2015). These results become only more discouraging when we look at the racial breakdown of the scores, and see that while 32% of White students and 47% of Asian students scored Proficient, only 7% of Black students and 12% of Hispanic students did. There is also a high correlation between Proficient scores and one or more parents having graduate from college (NCES, 2015). On the TIMSS, the United States faired slightly better, with 4th graders scoring 11th out of the 34 nations that participated, and 8th graders scoring 9th out of 33—but the nation still scored below Singapore, Korea, and China, and few U.S. students reached the highest-performing category on the exam.

These results suggest several things: one, that many students do not know as much mathematics as we would like them to; two, that there are significant achievement gaps between Black and Latino students, and White and Asian students, as well as between high- and low-income students; three, that math education is not making significant progress over time; and four, that the U.S. is measuring up only moderately well against competing nations.

Increasing math achievement for Black, Latino, and low-income students is particularly important so that we can begin to close gaps in post-K-12 success. Lee (2012) showed that students need to reach NAEP Proficiency in order to be prepared to attend a 4-year college, and that Black students' current NAEP scores suggest that they aren't prepared to enter even a 2-year college. It is not enough for just high-income, White students to achieve at high levels—or for them to be the only students exposed to high-level math teaching. In *The Hidden Curriculum,* Anyon found that students in the higher-tier "Executive Elite School", full of students with parents who are top corporation executives, math is about problem solving and reasoning. Students are asked to make decisions, justify their answers, and think of their own formulas (Anyon, 1980). I will argue later that this is exactly the way that math should, optimally, be taught; in 1980, however, this teaching ideology was reserved to the very wealthy, and data suggests that similar disparities continue today (interestingly, this article was written during the Back to Basics movement, a widespread attempt to *decrease* the achievement gap between low- and high-income students).

One more contemporary article echoing these ideas is Lubienski's (2012) analysis of Black-White achievement gaps on the NAEP, which breaks down student responses to statements such as: "There is only one way to solve a math problem" and "Learning mathematics is mostly memorizing facts", where agreement with these statements suggests that the students learn in classrooms that focus on rote learning (as exemplified in Anyon's low-income classrooms) rather than critical thinking (as in Executive Elite). Data was broken down by Black, Low Socioeconomic Status (SES); Black, High SES; White Low SES; and White High SES, and Lubienski discovered both that more Black students than White students reported agreement with both statements, and that there was only a small difference in agreement between Black, Low SES students and Black, High SES students. A larger percentage of Black, Low SES students always agreed with the statement than White, Low SES students. This study seems to disprove the idea that achievement gaps can be entirely explained by socioeconomic status—but then, what *does* explain it? The study suggests that even after NCTM published the 1989 *Standards* in an attempt to give all students access to the concept-based, critical thinking, it is still mostly white students who are experiencing the shift. Lubienski hypothesizes that the differences may be a result of lower teacher expectations for Black students; she also theorizes that there may be a cultural difference in student evaluations of the purpose of mathematics that teachers are then catering to.

These studies suggest several things: that high-income white students are faring better than low-income white students, that low-income white students are faring better than Black students of any SES, and that "faring better" on the NAEP, a test of problem-solving and application abilities, is correlated with a classroom that emphasizes reasoning and thinking over procedure and memorization. These differences matter, because unless classrooms change, Black, Latino, and low-income students will continue to be systematically disadvantaged when it comes to math achievement, diminishing their prospects of attending a four-year or any college, and therefore their prospects of acquiring a well-paying job (us Department of Labor).

Common Core: Ideology, Research, and Implementation

We cannot talk about the present situation of math education without discussing the implementation and possible impact of the 2010 Common Core State Standards in Mathematics (CCSSM), which have now been adopted by 42 states. CCSSM was introduced for multiple reasons: as a reaction to the U.S.'s unimpressive performance on the TIMSS, as well as to address achievement gaps between white students and underprivileged students of color; to move math education away from memorization and procedure and toward reasoning and critical thinking; and to make math standards more universal across the states, so that materials, assessments, and other supports can be distributed nationally (NCTM, 2010). It is this final purpose that makes CCSSM distinct from previous NCTM policies; it was the first that had the intention of making content more uniform across the us.

CCSSM retained the Standards for Mathematical Practice developed by NCTM in 2000, which urge teachers to help students be able to:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with Mathematics
5. use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Insofar as teacher practices, NCTM supplemented CCSSM with *Principles to Actions* in 2014, a document that sought to set forth "strongly recommended, research-informed actions" based on Common Core's principles. The document lays out eight Mathematics Teaching Practices, and discusses one in each section, providing the research that supports the practice, as well as examples of a teacher who is implementing the practice and one who is not. The practices encourage teachers to:

1. Establish mathematics goals to focus learning
2. Implement tasks that promote reasoning and problem solving
3. use and connect mathematical representations
4. Facilitate meaningful mathematical discourse
5. Pose purposeful questions
6. Build procedural fluency from conceptual understanding
7. Support productive struggle in learning mathematics
8. Elicit and use evidence of student thinking

The research cited in the document includes articles in cognitive psychology that provide explanations for why the practices should be helpful given what we know about how students learn mathematics, as well as classroom studies that explicitly show the increase in achievement that the practices elicit (Leinwand et al, 2014).

However, the major difference between CCSSM and previous standards was that CCSSM created strict requirements as to what content should be taught during what grade. This is because research conducted in preparation for the policy pointed out that what was considered to be “Proficient” varied widely by state to state according to their own individual assessments, and that the NAEP often painted a much different picture than what state assessments would suggest. Furthermore, the TIMSS also showed that countries with content standards that were characterized by their Focus, Rigor, and Coherence tended to outperform countries, such as the us, with content standards that were “a mile wide and an inch deep” (McDonnell & Weatherford, 2015), so it made sense to change the content standards in the U.S. so that they mimicked the supposedly superior model.

However, those who formed this interpretation were those who already supported CCSSM—whereas other researchers interpreted the U.S.’s lack of success to mean that the previous standards were poorly implemented, because curriculum materials and teacher training were not helping teachers to bring their students to the proficient level. By this interpretation, common standards across states was “necessary but not sufficient” (McDonnell & Weatherford, 2015), and other changes needed to be made before we would see a significant change.

In terms of the development of the standards, The Council of Chief State School Organizers (CCSSO) and the National Governors Association (NGA) asserted that they would base their standards off evidence, rather than personal judgment, as had happened in the past; they aimed to connect what happens in the K-12 classroom with what students will need for higher education and future careers. In accordance with these aims, standards used the research base of developmental psychology to decide on trajectories for K-2. Since such research as not as concrete for older children, for the higher grades they instead relied on math education researchers' beliefs about how students learn, and mathematicians' beliefs about the logic of math as a discipline to come to their conclusions. In this way, the writers of CCSSM did indeed work to involve research and evidence in the writing of the standards, when it was available.

But as with previous reforms, and as some researchers predicted, CCSSM struggled with issues of implementation. Having content standards weren't enough, as teachers needed to have the necessary understanding to teach to the standards, and also the proper materials. One issue of implementation became clear when a Facebook post "went viral" that was shockingly similar to James Shackelford's critique of New Math. In the post, a father took a picture of his elementary school child's math homework, which involved correcting the mistake an imaginary student had made while using a number line to complete a subtraction problem. The father argued that it was a convoluted problem much too difficult for him to even understand, even though he had a "Bachelor of Science Degree in Electronics Engineering" (Klein, 2014). Anyone looking at the problem would agree that it was unnecessarily complex, and it caused many to argue that CCSSM was fatally flawed. However, CCSSM is not a set of worksheets or practice problems, but a set of standards for practice and content. Because of this, it has been difficult for teachers to adopt the practices that Common Core asks for, to learn the content they are required to teach, and to locate materials that align with what CCSSM actually suggests (Zhang, 2016; Stern, 2016). Furthermore, it is often the case that teachers do not support or understand the changes that CCSSM seeks to create, making them resistant to teaching it (Stern, 2016). These realities suggest that in the future, NCTM should focus less on developing standards for different grade levels and more on ensuring that teachers support and have the materials and knowledge to implement the standards successfully. Furthermore, teachers need to know what types of practices will lead students to acquire the skills laid out in the CCSSM Process Standards. In the following section, I discuss classroom studies which provide suggestions for what these practices may be.

Conclusion: Answering the Research Questions

Having completed my analysis of CCSSM, I pose my questions once more: According to the writers of Common Core, what teaching practices can math teachers employ that increase student achievement *and* engagement in math? And furthermore, which of these practices can also help to decrease achievement gaps between students of different racial and socioeconomic groups?

Most of the explicit practices recommended by NCTM came to light not in Common Core itself but in the supplementary *Principles to Actions*. This document discussed research-supported actions that teachers can take to help their students come to a stronger understanding of mathematics and achieve at higher levels (Leinwand et al, 2014). Many of these actions have also been shown to decrease achievement gaps between students of different racial backgrounds (Boaler & Staples, 2008; Boaler, 2006). I have included the actions once again below:

1. *Establish mathematics goals to focus learning*
2. *Implement tasks that promote reasoning and problem solving*
3. *use and connect mathematical representations*
4. *Facilitate meaningful mathematical discourse*
5. *Pose purposeful questions*
6. *Build procedural fluency from conceptual understanding*
7. *Support productive struggle in learning mathematics*
8. *Elicit and use evidence of student thinking*

Recent Research: Classroom Studies

Introduction

In this section, I will discuss some of the teacher practices listed above, synthesizing studies that have attempted to gauge their affect both on achievement, and on students' ideas concerning what mathematics is about. It accordance with cognitive psychology, it only makes sense that interventions which improve achievement also should result in students conceptualizing mathematics in a way that aligns more closely with what working mathematicians consider their discipline to be about, i.e. reasoning precisely, understanding concepts deeply, and applying ideas to new situations, as well as what cognitive psychologists suggest will help students understand material best.

I want to emphasize that none of these studies are meant to recommend the "number one method" for raising achievement or increasing student engagement. In fact, due to many variables, it is often hard to say what, exactly, is the key factor influencing achievement; a classroom experiencing concept-based instruction may actually simply be benefitting from an understanding teacher who knows how to build off students' prior knowledge. Or perhaps students in a traditional classroom are simply struggling because their teacher doesn't know enough mathematics. Furthermore, many studies involve comparing a "traditional" classroom with a "reform-oriented" classroom that is different in a multitude of ways, making it unclear which component is causing the change in achievement. But I don't think this is a drawback of the research, so much as a reminder that many variables come together to give students a positive educational experience. In particular, it is vital that classrooms include significant, worthwhile content, that they call for intellectual rigor, that they provide equal access to students of differing ability levels, that teachers ask students the right questions, and that they give appropriate and clear explanations when necessary (Weiss & Pasley, 2004). A reform-oriented classroom that does not include a variety of these factors will probably fail, and a traditional (perhaps lecture-based) classroom that *does* include them will probably succeed. Of course, the tendency seems to be for the reform-oriented classrooms to include these practices, and for traditional classrooms to lack them.

It is also important to note that the studies described below discuss students whose achievement in math is not only gauged by their performance on procedure-based, multiple choice tests, but rather on tests that assess their ability to reason with and apply mathematics in new situations. This applies both to the tests given by researchers as well as to the various standardized tests discussed, such as the GCSE. Furthermore, it is often shown that those students who acquire the abilities listed above (understanding, reasoning, and applying) are also able to complete more procedural problems that don't necessarily require these skills, even if they didn't learn the procedures explicitly.

Growth vs. Fixed Mindset & Heterogeneous Grouping

The first of these ideas in mathematics education is more of a concept than an approach or practice, and that is the concept of *Growth vs. Fixed Mindsets.* Introduced by Psychologist Carol Dweck in her 2006 book *Mindset: the new psychology of success,* the concept of Growth and Fixed Mindsets has helped U.S. to understand the very different ways in which students approach education, including mathematics. Students with a Growth Mindset believe that intelligence is something that can be learned, that through practice and study a person can become smarter and increase their mathematical abilities. Someone with a Fixed Mindset on the other hand believes that intelligence is static. In their minds, some people are good at math, and some people are not. Failure is a result of their innate inability to do mathematics, and higher-level mathematics is repelling to them because they don't believe they are capable of understanding it (Dweck, 2006; Boaler, 2013).

A concept that is often linked to Fixed Mindsets is *stereotype threat*; this is the idea that when there is stereotype about a group of people, like that Black people aren't good at math, that stereotype can actually affect those students' performance because they believe it to be true. In other words, these groups may have Fixed Mindsets around intelligence, believing perhaps even subconsciously that people of their race or gender are not as "smart" as people from other groups, and there isn't anything they can do to change that. Aronson et al (2003) found that an intelligence mindset intervention actually helped to increase Black students' grades and their enjoyment at school. In the study, Black and White students participated in three sessions in which they wrote letters to "at-risk seventh graders" convincing them that they would be able to succeed despite their struggles. Students learned of research concerning the malleability of intelligence and were encouraged to include this research in their letters. The researchers found that the sessions did in fact have a significant affect on student achievement and enjoyment of academics in comparison to the control, and that there was more of an effect on Black students than on White students.

Studies like this one suggest that stereotype threat is real, and that perhaps Black students are more likely than White students to have a Fixed Mindset that is hindering their ability to perform up to their potential. Unfortunately, the widespread practice of Ability Grouping or

'Tracking' in U.S. mathematics education suggests to students that intelligence *is* Fixed, that if you are placed in the lower track you are innately less intelligent than those on the higher track and cannot change that. According to Boaler (2013) this tracking decreases the performance of *all* students.

Fortunately, Boaler conducted another study on the effect of a practice that could be seen as the antithesis to tracking: *Heterogeneous Grouping*. As one could assume from its name, heterogeneous grouping is a practice in which students in a classroom, mixed or not, work in groups with students of a variety of ability levels on open-ended problems. In this particular study, the school attempted to reduce some difficulties associated with heterogeneous grouping (imbalance of work, students being excluded) by using an approach called *complex instruction.* This type of instruction includes the use of four practices. The first is emphasis on *multidimensionality,* the idea that math involves many different types of tasks that require different abilities, and no one will have all of them, but everyone will have at least one. The second is the use of *roles,* in which students take on different roles with different responsibilities, such as "facilitator, team captain, recorder, or reporter", encouraging students to work together and rely on each other (Boaler, 2006). The third is *assigning competence,* in which teachers bring attention to the intellectual contributions of low-status students (perhaps those who are low-performing or shy) in order to raise their status. The fourth is *student responsibility,* a practice that encourages students to make sure that the other members of their groups also understand the material. Teachers do this by giving group tests, and by randomly choosing a student from each group to answer a question, making it necessary that all students understand the material just in case they are chosen (Boaler, 2006).

The approach is grounded in the idea of creating an equitable classroom, in which students see all other students as being intelligent and having something to offer. The approach also treats intelligence as malleable and complex; *multidimensionality* seeks to move away from the idea that if you can't memorize formulas you cannot do math; *assigning competence* supports students who tend to struggle, and views them as valuable possessors of knowledge. The fact that students are both in a mixed-ability classroom with mixed-ability grouping and encouraged to see each other as intelligent and valuable certainly promotes the idea of a Growth Mindset, suggesting that students would see an increase in achievement.

This idea of *complex instruction* was part of a larger study, in which Boaler compared Railside School, which had just instituted these reforms, with two more traditional schools that had more White and middle-class students (Railside was largely Black and Latino, and low-income). The researchers gave the California Standards test to students from all three schools before the study, after the first year, and after the second year, and found that while in the pre-test, Railside students lagged behind their peers at the other schools ($p=.014$), by the end of the second year they were performing significantly better ($p=0.000$). The study also found that, as with the study by Aronson et al, significant differences between White, Black, and Latinx students disappeared by the end of the second year (Boaler & Staples, 2008). Disparities between students of different races persisted at the traditional schools.

On a more qualitative note, students at Railside had significantly more positive views on mathematics than students at the traditional schools. 71% of students at Railside said that they enjoyed math class, 25% more than those in traditional classes (Boaler & Staples, 2008). Furthermore, the study found that students at Railside were more interested in mathematics, believed they had more authority to validate mathematical ideas (as opposed to a teacher or text), and believed they had more agency in mathematics, where agency is defined as "having the opportunity to inquire and use their own ideas" (Boaler & Staples, 2008). Furthermore, all students at Railside said they planned to take more math courses in the future, and 39% said they wanted to pursue a future in mathematics, compared to just 5% of students in traditional classes.

This study both refutes the idea that students perform better when they are tracked into different groups, provides further evidence for the value of a Growth-Mindset, and suggests that heterogeneous grouping, when implemented correctly, can be a valuable approach to learning. Not only did the approach increase student average achievement, it also eliminated racial achievement gaps, and resulted in students liking math more and being more interested in it, with plans to pursue it in the future. Whether educators' goals are to increase the number of students in STEM, increase achievement, fight for social justice, or just make their students more invested, it seems clear that heterogeneous grouping and *complex instruction* are valuable tactics. Furthermore, through encouraging student small-group discussion of interesting problems, these practices align with the NCTM-recommended practices of *Facilitate meaningful mathematical discourse,* and *Elicit and use evidence of student thinking.*

Project-Based Learning & Active Engagement

Another study conducted by Boaler at Phoenix Park (1998) reinforces the benefits of focusing instruction on group work rather than lecture in the classroom—but while the classrooms do use heterogeneous grouping, what is more interesting is the type of work students are doing in their groups. The study compares the math achievement and perceptions of students who learned math in an open environment, where students worked on open-ended projects that forced them to apply mathematic concepts to real situations, and in a closed environment, where students listened to lecture and completed exercises in a textbook. Phoenix Park stood as the model for open mathematics, and Amber Hill the model for closed mathematics; both schools were located in working-class neighborhoods in England, and both were middle schools, with students having attended the same elementary school in the years before the study began. In open or "process-based" mathematics, students must "make their own decisions, plan their own routes through tasks, choose methods, and apply their mathematical knowledge" (Boaler, 1998), a list of requirements that align closely with the practices *Implement tasks that promote reasoning and problem solving* and *Support productive struggle in learning mathematics.*

Proponents of this type of mathematics teaching argue that it increases student enjoyment and understanding, decreases achievement gaps between genders, and enhances transfer to new situations, including 'real life'. Closed mathematics, on the other hand, is hypothesized to have the opposite effect, making it difficult for students to apply their largely procedural knowledge in a meaningful way.

A statement from the Head of Mathematics at Amber Hill synthesizes the results of this procedural practice quite clearly:

> Students are generally good, unless a question is slightly different to what they are used to, or if they are asked to do something after a time lapse, if a question is written in words, or if they are expected to answer in words. If you look at the question and tell them that it's basically asking them to multiply 86 by 32, or something, they can do it, but otherwise they just look at the question and go blank.

The Amber Hill students show traits that are antithetical to the qualities we want to encourage among math students. This paragraph, and preceding descriptions, show that students at Amber Hill are able to make calculations when the procedure is clear to them, but as soon as problems concern real-world objects, contain words, or are "slightly different to what they are used to", students shut down. Given this disconnect, it seems unlikely that the abilities that the Amber Hill students possess will ever be useful to them, whether at home or in the workplace, and it is further unlikely that these skills will be useful if they pursue mathematics as a discipline (the study finds that only 6% plan to do so).

At Phoenix Park, on the other hand, students would be presented with an idea or problem at the beginning of a class, and were encouraged to pursue the problem from any angle they would like for three weeks. As they worked, teachers would give instruction to different groups of students as necessary, depending on what they were struggling with. Because the teacher was often occupied with classmates, students knew that they would need to work independently, and often did so, working through problems rather than immediately seeking help.

In terms of quantitative achievement, the study found that at Phoenix Park, significantly more students scored a passing grade on the General Certificate of Secondary Education (GCSE) Exam; the exam often determines what sixth form (school for ages 16-19) students in the UK are able to attend. Boaler writes that the exam had a ratio of procedural to conceptual questions of 2:1 (Boaler, 1998). While the teachers at Amber Hill felt that they were teaching to prepare students to pass this test, those at Phoenix Park did not review for the exam. Regardless, the study found that while similar percentages of students at the schools scored A*-C on the GCSE, a significantly larger percentages scored A*-G at Phoenix Park, with G being the lowest passing grade. This is regardless of the fact that Phoenix Park students didn't complete explicit review for the exam, rarely completed procedural questions in class, and did not cover all of the math objectives and topics on the exam. This fits my hypothesis that mathematical thinking abilities can compensate for a lack of practice with procedural questions.

The study also found that students who learned through the project-based method at Phoenix Park remembered significantly more of the material six months later than students at Amber Hill. The study also compared student answers on a post-test and delayed post-test, where they found that among those in Year 10, Amber Hill students answered 50% of questions correctly on the delayed test that they answered correctly on the original, whereas Phoenix Park students answered 83% of the questions correctly. The low answer rate of Amber Hill students emphasizes that oftentimes, the procedural information learned and memorized in school is quickly forgotten. But the Phoenix Park students, in addition to remembering more, have more than just procedures. They have learned how to think and reason mathematically, how to problem solve, and how to be independent thinkers, and it seems likely that these skills will stay with them whether they remember the mathematics they learned or not.

Active and Passive Engagement, and Future Math Achievement & Mindset

In fact, Boaler and Selling (2017) Conducted a follow up study almost 20 years after their initial article on “Open and Closed Mathematics”, in which they talked to many of the students who attended Phoenix Park and Amber Hill, analyzed employment and socioeconomic data, and asked them how they feel about math and how it factors into their adult lives. In this article, the authors conceptualize *active and passive engagement* in mathematics, where *active engagement* occurs when students are “engaged in problem solving, the discussion of ideas, and the application of methods” (Boaler & Selling, 2017). *Passive engagement* occurs when students are largely relying on teacher explanations and simply reproducing what they learn from lecture. Hatano & Oura (2003) might further suggest that students who are actively engaged in mathematics acquire *adaptive expertise* of the field; they are able to: "apply their schemas in more adaptive and tuned ways (Lesgold, Glaser, Rubinson, Klopfer, Feltovich, & Wang, 1988). They may understand why their procedures work, modify known procedures, or even invent new procedures (Hatano, 1982). They may respond quite flexibly to contextual variations. They may also be able to cross a boundary between domains to find better solutions (Engeström, Engeström, & Kdärkkäinen, 1995)" (Hatano & Oura, 2003). These abilities match well with the abilities of the students at Phoenix Park, who were well-practiced at applying mathematical concepts to whatever situation they encountered.

Studies suggest that students who are actively engaged in mathematics learning tend to develop a positive and engaged mathematics identity, whereas students who are passively engaged in mathematics tend to get frustrated. Especially for adolescents, who are striving to become more autonomous and make their own decisions, feeling that they are confined to prescribed methods for finding answers may create resistance (Boaler & Selling, 2017).

Given that this was a follow-up study, not all initial participants responded, but Boaler & Selling were careful to consider the differences in socioeconomic status and GCSE scores between initial participants and later respondents. They found no significant difference in the former for either group, and found that for Amber Hill, respondents had higher GCSE scores than nonrespondents, but the researchers excluded a few respondents from their interviews to balance this out. The study found that 65% of Phoenix Park students had experienced upward social mobility, while only 23% of Amber Hill students had; 51% of Amber Hill students had jobs in a lower classification than their parents. These results could suggest that closed-math classes are correlated with downward mobility, though given the progressive nature of Phoenix

Park as a whole, it seems likely that more than one subject experience combined to give Phoenix Park students an advantage.

Boaler & Selling also conducted interviews with the participants, in which they asked them questions about their math learning in school, and how they use math today. Their responses on the prior question were characteristic of what one might expect; Amber Hill students recalled math as being about memorization and procedure, and being very distant from real life; they wished it had been different, and often expressed that they enjoyed math much more today. Phoenix Park students remembered being asked to think and work on interesting projects, and that they enjoyed math. More interesting, however, were the participants' responses when asked if and how they used math in their lives today. Participants who'd attended Amber Hill focused on content, mentioning subjects like trigonometry, and saying that they couldn't remember it anymore and that they didn't use it. Phoenix Park students, on the other hand, approached their characterizations of math in a different way; they said that they *did* use math in their day-to-day lives, but they didn't point to particular topics or subjects. Rather, they spoke of problem solving and logic, saying that their math education helped them to work through problems they experienced, and reminded them that there could be more than one way to work it out. Participants from Phoenix Park enjoyed being given responsibility and problem solving in the workplace, whereas participants from Amber Hill students said that they would refer to a superior if they were struggling.

The quantitative and qualitative data provided by the study suggest firstly that progressive education, including progressive, open, math education, may be correlated with upward mobility and the ability of students to get skilled and highly skilled jobs. Furthermore, interview data makes it clear that Phoenix Park students' experience being actively engaged in an open math classroom caused them to see math as less about content and procedure and more about problem solving and critical thinking, and that these skills carried into their futures, giving them more autonomy in the work place and more confidence when it came to solving problems related to (and unrelated to) math.

Conclusion

I believe that my findings in terms of classroom practices can be separated into two sections: math mindsets and orientations that we want our students to have (and others we want to avoid), and the practices that encourage these mindsets and orientations. We want students to have a Growth- rather than Fixed-Mindset; Railside school shows U.S. that this can be achieved through detracking and complex instruction, which includes heterogeneous grouping as part of its method. We want students to be actively engaged in mathematics, and to have *adaptive* rather than *passive expertise;* Phoenix Park tells U.S. that project-based learning can assist us. We want students to have these Mindsets because they are linked to the practice of truly doing mathematics, and because, naturally, students who have these mindsets, who learn math in these ways, both perform better than more traditionally schooled peers on standardized and more tailored exams, and because they cause students to enjoy math more, to see its relevance to their daily lives, and possibly even increase their socioeconomic status. Furthermore, the study of Railside shows that complex instruction not only raised achievement but eliminated achievement gaps between White, Black, and Latino students.

Given these results, it is perhaps surprising that math is still taught in such a procedural way; interviews with teachers and students at Amber Hill suggest this is because teachers feel they must "teach to the test", and that the tests are largely procedural in nature. However, it was clear that students at Phoenix Park, despite hardly preparing for the GCSE whatsoever, did as well as if not better than their peers at Amber Hill. Which makes sense, given that mathematics is about critical thinking, and Amber Hill students rarely engaged in anything of the sort. But I doubt this is the only reason; Liping Ma (2010) suggests that perhaps the reason so many U.S. teachers provide such procedural instruction is because they themselves do not understand concepts well enough to explain them. Which makes sense, given that they learned math in a system were concepts were so often deprioritized in favor of the memorization of facts.

Then what is the solution? While it is unclear what percentage of math classrooms still contain procedural, didactic teaching, the number is certainly large. Even those teachers trying to implement the Standards as provided by NCTM struggle to do so, lacking true understanding of the necessary concepts and teaching ideologies. An analysis of the history of math education in the U.S. made it clear that no new set of standards or principles will succeed unless schools and classroom teachers support them and know how to teach with them in mind. Although research is far from complete, we currently have a wealth of valuable research on how to teach mathematics well. We now need more research on how to transfer that knowledge to classrooms, so that all students, of all genders and races, have equitable opportunities to succeed in learning mathematics, and access the many advantages that this success can make available.

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