Revision and the Realm of Actuality: 
Problematizing Wittgenstein’s Account of Logic

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Senior ThesisSubmitted in partial fulfillment of the requirements for receiving the Degree in Bachelor of Arts in Philosophy at Haverford College

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April 22nd, 2016
Acknowledgements

There are many to whom I am indebted to in regards to this paper. I’d like to start by thanking
the philosophy department and all of the professors I’ve taken courses with for always taking me
seriously as a scholar and giving me endless opportunities to grow.

I’d like to thank some of my fellow philosophy students who have spent hours on end not only
talking about our theses but talking about philosophy in general. Thank you to Griffin Stevens,
Andrew Dalke, Dylan Reichman, Baptiste Teyssier, and Luke Jensen. All of you have inspired
me to work harder than I ever thought possible. Your encouragement, support, and constant
willingness to engage in philosophical discussions has made me a better, more confident
philosopher.

I’d like to thank my second reader, Dr. Joel Yurdin, whose unmatched pedagogy and continuous
support helped me narrow down my topic and add challenging and interesting content to my
paper. My discussions of Aristotle would be entirely lacking without his incredible instruction on
the matter.

I’d like to thank my first reader, Dr. Danielle Macbeth, whose own work has been profoundly
influential on my own thinking. Professor Macbeth has always inspired me to love and pursue
philosophy and has always encouraged me to work as hard as possible. Without having had taken
her courses Historical Introduction to Logic, Philosophy of Mathematics, and Analytic
Philosophy of Language, I would have never found my topic or developed my deep interest in
logical revisability.

Finally, I’d like to thank my mother for doing what I still cannot do after four years of college –
proof reading my own paper.
Abstract:

The project of this paper is to problematize the Wittgensteinian account of logic which treats what we understand to be the laws of logic as infallible and claims that logic does not constitute a branch of knowledge. In his essay “Two Dogmas of Empiricism,” Quine takes a radically different position; he argues that all of our beliefs are revisable, including what we believe to be logical laws. In this paper I will argue in favor of logical revisability – citing three instances that I believe demonstrate why we have and why we are able to revise our system of logic (the three instances being: the inconsistency between syllogistic and first-order logic, the evaluative nature of the accepting or rejecting certain laws of logic, and finally, how logical notation mediates our access to truth). By first establishing what I believe to be a shared characterization of what logic is, I will then follow a Wittgensteinian course of reasoning to expose the assumptions that Wittgenstein mistakes as universal truths – specifically that there is no difference between “the laws of logic” and “what we understand to be the laws of logic”. This discussion will make space for considering how and when one revises the laws of logic as well as the relationship between our systems of logic and what we perceive as the realm of actuality evolving – both of which will call for a reconceptualization of logic as a branch of knowledge.
I. Introduction

There is a divide among philosophers concerning whether logic could be considered a branch of knowledge. Philosophers who don’t believe that logic is a branch of knowledge take the position that logic is not something we can study – there isn’t anything about logic to be learned. These philosophers believe that what we understand to be the laws of logic are infallible and being infallible therefore have no content that needs to be studied. This position, which has been taken by most philosophers in the history of logic, finds its contemporary bearings in Ludwig Wittgenstein’s *Tractatus Logico-Philosophicus* where Wittgenstein characterizes logic as a merely formal practice that neither extends our knowledge nor tells us anything about the world.

Alternatively, one can take the positions of other thinkers, such as Willard Van Orman Quine or Gottlob Frege, who treat logic as a science. Logic entails studying the nature of our reasoning, specifically deduction, where one is concerned entirely with what logically necessarily follows from given propositions. This view of logic incorporates what the Wittgensteinian view ignores: we can always refer back to our capacity to reason when our formulations of the laws of logic do not meet the current demands of our intellectual practices. Despite what the Wittgensteinian may claim, the study of logic has a history in which our characterization of logical laws have responded to advancements and changes in how we understand the world and access knowledge.

As a result of the ever-evolving demands of our intellectual practices, logic has been transformed – or “revised.” In his essay “Two Dogmas of Empiricism,” Quine claims that our knowledge is like a web where the more fundamental beliefs are at the center and less central beliefs, which tend to be our more empirical ones, are towards the edge. At the edge of the web,
recalcitrant experiences are able to alter our beliefs. Some experiences are so disruptive that they can alter beliefs deeply imbedded in the web: the more contrary to our current belief system the experience is, the greater the change that is made to the web. As a result of understanding our knowledge as constituting a web, Quine takes the radical position that everything we consider to be knowledge is empirically revisable. Although we don’t want to take the position that any empirical evidence could revise logic (we will see rather quickly that logic isn’t justified empirically therefore cannot be rejected empirically),¹ we should take very seriously the idea that it is possible to imagine that when large portions of the web are called into question so that even the core of the web is altered, what we understand to be the laws of logic are revised. When the web undergoes a significant disruption (such as a scientific revolution) we revise our laws of logic.

It is for these reasons that one shouldn’t accept logic as infallible. We have, and always will have, the capacity to revise logical principles due to what is intelligible to us (or as what we understand the realm of actuality to be) changing. In this paper, I will defend the Quinean belief that logic is revisable against the Wittgensteinian position by demonstrating that our logical laws have already undergone revisions as a result of revolutions in how we conceive the world and that the possibility of further revisions will always exist as a result of how logic mediates our access to truth.

¹ Quine too believes that it would be misguided to claim as others have (such as Putnam) that as a result of empirical evidence one is licensed to revise the laws of logic. He writes in another paper: “I am committed to urge the empirical nature of logic and mathematics no more than the unempirical character of theoretical physics; it is rather their kinship I am urging, and a doctrine of gradualism” (1970, 100).
II. Characterizing Logic

First, I shall give a characterization of logic that holds true regardless of whether or not one considers logic to be a branch of knowledge. The field of logic is related to reason, or as Graham Priest writes: “the canonical application of logic is to reasoning”. Logic studies rules that license us to perform inferences such that the truth-value of the conclusion is guaranteed. Thus a characterization of logic will be a characterization of how one reasons correctly (or how one reasons when they are successfully exercising their capacity to do so).

Our formulations of the laws of logic do not act as guidelines that one chooses to follow; they are necessarily followed when one is actualizing their capacity to reason. In his lectures on logic, Immanuel Kant asserts: “like all our powers, the understanding in particular is bound in its acts to rules we can investigate”. Our capacity to reason is a capacity (or “power”) to be exercised, much like one exercises a muscle in a sport. When one is exercising that capacity, they are bound by the laws of logic. If one violates a law of logic, they are no longer exercising their capacity to reason. Just as an illusion may have the same phenomenological character as perception despite the actor not perceiving, so too can thinking illogically (such as improperly performing a logical inference) have the same phenomenological character as reasoning while failing to actualize the capacity. Whenever one is exercising their faculty of understanding, they are doing so within the confines of logical laws.

When adhering to the laws of logic, one preserves truth-value. Thus, from true premises, one can deduce logically necessary conclusions. Note that this is opposed to necessary conclusions. For example, if a ball is blue, the ball is not red. There is nothing illogical about the

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4 I was referred to this example by Danielle Macbeth.
ball being both blue and red, rather it is a physical necessity that the ball is only one color. It is a contingent truth. While a proposition may present a thought that violates a law of necessity and remains understandable, the same cannot be said for something that violates the laws of logic. A condition for a proposition to be a proposition (to express something meaningful) is that it does not violate logical necessity. One can understand the sentence “the ball is a blue thing and the ball is a red thing” despite recognizing that it is necessarily not true. One cannot, however, understand “the ball is a blue thing and the ball is not a blue thing”. This doesn’t mean that the premise is illegible – we can still read it – but it cannot be considered seriously.

What we understand to be the laws of logic appear to us as self-evident – meaning that these laws do not require a demonstration to determine that they are correct. Aristotle proposed that what we take to be the most basic laws of logic, such as the law of non-contradiction (it is not the case that $p$ and $\neg p$) and the law of identity ($p$ is equivalent to $p$), are so fundamental that they take absolutely no work at all to derive. Our slightly more complex laws, such as modus ponens or modus tolens ($p \supset q$, $p$, so $q$, or $p \supset q$, $\neg q$, so $\neg p$)\(^5\) may appear to be less evident to us, but this is due only to their complexity. One could use truth tables to demonstrate how the truth-values of either $p$ or $q$ can impact the truth value of the entire proposition. Other laws, such as DeMorgan’s law ($\neg (p \land q) = \neg p \lor \neg q$ or $\neg (p \lor q) = \neg p \land \neg q$), seem intuitively true but take quite a bit of work to rigorously prove.\(^6\)

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\(^5\) There are two things I’d like to note about the formulation of these rules of inference: the sign “$\supset$” represents a relation similar to “if... then...” yet is not identical and that I don’t claim this to be the pure way of representing these rules. Firstly, when using logical notation, one doesn’t use if... then... to express the relation because the nature of natural language is such that there is vagueness where in logic the relations need to be rigorously defined. Secondly, I merely use this figuration of the laws of logic to get across a general idea of the complexity of our contemporary conception of these laws.

\(^6\) Rigorous meaning that every step is licensed by a rule stated in advance. This is a requirement upheld by mathematical logicians.
The illogical is that which violates the laws of logic and is something we don’t consider as ever actualizing. Take, for example the following proposition “this animal is a goat and not a goat”. This proposition violates the law of non-contradiction. When we actualize our ability to reason, we are unable to think the sentence for one cannot think in any fully aware state of an animal that is both a goat and not a goat. It is difficult to say much more about what the illogical is doing – which speaks to the difficulty of grasping its character.

Given these facts, we can understand why one might adopt the Wittgensteinian view. It’s difficult to imagine how one might propose to revise laws that appear to us as self-evident. Further, any type of deduction done that violates these laws will result in a contradiction, something that we can’t at this moment think or consider as a serious possibility. It then seems that logical laws aren’t really laws at all. Typically, the act of defining a law expresses how things ought to be or how a process ought to work. For example, in physics we discover laws of motion that govern how things in the world move. We have to derive these laws because it’s not self-evident how things ought to be. As a result of them not being self-evident, we are also capable of understanding their negation or revision at any given moment. We can also make mistakes in physics. For example, we may not have equipment capable of measuring the speed of light accurately enough so when we estimate how long it will take a certain beam to reach a certain space, our prediction could be wrong. But in logic, we never have this problem because the “laws” aren’t something we can so naturally consider to be incorrect. Also, whenever we use rules of inference, the conclusion is logically necessary. Thus, for the Wittgensteinian, the “laws” of logic aren’t actually laws. They don’t say anything about the world. They merely demonstrate the realm of possibility and that realm of possibility exhausts reality. Anything that
violates the laws of logic (a contradiction, an incorrect deduction, etc.) produces nonsense\textsuperscript{7}, thus demonstrating that nothing can be constructed outside of the laws of logic.

Yet, I believe that the Wittgensteinian takes as blind truth the assumption that “the laws of logic” and “our formulation of the laws of logic” are equivalent. The former refers to the actual principles that dictate reason and preserve truth-values while the latter refers to what we as finite beings have access to. Before arguing why I believe that we must maintain the distinction, I will give a fuller picture of the Wittgensteinian position as well as argue what some of the consequences are of this position.

\textbf{III. The Wittgensteinian View}

In this section, I will demonstrate why Wittgenstein’s assumptions about the nature of logic lead him to characterize a godless logic such that (what we understand to be) the laws of logic can never \textit{laid down} or \textit{determined}. They don’t function as laws in any important sense (they have no content or knowledge that they express). The first assumption Wittgenstein treats as compulsory is that there is only empirical content and logical form. This assumption will lead us to the conclusion that (what we believe to be) the realm of actuality never changes. Logic thus doesn’t study any content and is not to be considered a branch of knowledge.

For Wittgenstein “the world is all that is the case”,\textsuperscript{8} meaning that the only content that exists is empirical. Being that logical laws are a priori, logic is reduced to being a strictly formal practice with no content that merely makes explicit the realm of actuality. The laws of logic then must be unrevisable because they have no content to revise. If they were to be revised, then what

\textsuperscript{7} I use nonsense here not in the classical Wittgensteinian sense, but rather to describe propositions that are not comprehensible.

could be made explicit from propositions, i.e. what was already implicit in propositions, would change. The laws would also be expressing something outside of the realm of actuality – a non-empirical content – and the realm of actuality would change. Yet, this would imply that there is inaccessible, non-empirical content that can be rendered intelligible, violating the dichotomy Wittgenstein accepts (that there is only empirical content and logical form). This position also forces us to claim that the realm of actuality can never change. If the realm of actuality were to change, there would need to be another realm into which the bounds of the actual could expand and vice versa (a non-empirical content). Yet, there is no such realm for Wittgenstein. What is intelligible to us (what has content) has always been the same set of things.

For Wittgenstein, the world is the “totality of facts” (1921, 1.1) and these facts are expressed by propositions. He defines a proposition as something that “communicates a situation to us, and so it must be essentially connected with the situation” (1921, 4.03) and that “the connexion is precisely that it is its logical picture” (1921, 4.03). Thus, according to Wittgenstein, the world is the totality of facts expressed as propositions and the laws of logic are the conditions for the propositions to express anything meaningful because they are the underlying structure of the realm of actuality. Further, for a proposition to express the states of affairs, it must have a sense (show a picture of a situation (Wittgenstein 1921, 4.031)) and this sense is the proposition’s “agreement and disagreement with possibilities of existence and non-existence of states of affairs” (Wittgenstein 1921, 4.2). Thus a proposition either has a truth-value of true or false: true if it agrees with the states of affairs and false if it does not.

God (an infinitely powerful being) is unable to govern (what we understand to be) the laws of logic because there is nothing to govern. Wittgenstein writes: “it used to be said that God could create anything except what would be contrary to the laws of logic. – The truth is that we
could not say what an ‘illogical’ world would look like” (1921, 3.031). By “could not say”, Wittgenstein means that we don’t have the capacity to put into propositions what an illogical world would look like. One may be able to piece together words and form an illogical sentence, but in this case one has failed to actually form a proposition (i.e. form a sentence that can agree or disagree with the states of affairs). Thus the illogical world is impossible to postulate about (I will revisit the exact consequences of taking an illogical sentence seriously shortly). God has the ability to rewrite the laws of physics such that rate of speed is no longer equivalent to distance traveled over total time travelling or the laws of mathematics such that the angles of a triangle do not add up to 180 degrees. We also have the ability to postulate about what this world would look like (as demonstrated by our ability to take the previous sentence seriously). God cannot, however, make it such that \( p \land \sim p \) (a contradiction) is the case. In fact, God cannot even make \( \sim (p \land \sim p) \) (law of non-contradiction) the case. Logical laws aren’t even the kind of things that are “made” – rather they are the conditions for the actualization of a state of affairs.

If God laid down the laws of logic, he would be determining content and therefore saying something about the world. If one is able to say something, i.e. express content, then one is able to conceive of the negation of that content, thus making space for the illogical. For Wittgenstein, however, logic doesn’t have any content; there is nothing to negate. Logical laws only help us make explicit what was already implicit in the picturing of a state of affairs. If they were to say something, we would then be drawing the boundaries of the realm of the actual. Defining the realm would imply that there is a space outside of actuality. This space would contain the impossible- i.e. the things that one cannot actualize. Yet, as Wittgenstein has already made clear, the world is all that is the case; there is no realm outside of the realm of actuality.
For Wittgenstein we cannot postulate about the illogical (postulate about the realm of impossibility), because of the role that propositions play in representing reality. Every proposition, in virtue of being a proposition, can be rewritten into a logical notation, i.e. written using logical connectives and structure. Because every proposition necessarily has a sense and this sense is the logical picture of reality (therefore can be expressed in logical form), translating a proposition into logical notation is merely formal. Then, if we want to be able to say something about logic itself (say something about the realm of actuality), we must be able write a proposition about it, much like we do for physics or math. Wittgenstein writes: “in order to be able to represent logical form, we should have to be able to station ourselves with propositions somewhere outside logic, that is to say outside the world” (1921, 4.12). This is impossible, however, because anything formed outside of logical form is not a proposition (cannot agree or disagree with the states of affairs) thus preventing us from ever postulating while stationed outside of logic.

But what if we were to try and take seriously a contradiction (the illogical)? Quine defines the consequences taking of a contradiction seriously as follows: the “acceptance of one sentence and its negation as true would commit us to accepting every sentence as true, and thus forfeiting all distinction between true and false”. By destroying the distinction between true and false, we are no longer representing the realm of actuality. For example, if a student asks her friend advice on what her options are in regards to finishing her essay and her friend responds to her by saying, “you write your essay and you do not write your essay”, the friend has not actually given any advice. She hasn’t carved up the realm of actuality in order to tell her friend how to act; instead she has formed a sentence that the student cannot consider seriously (she cannot

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entertain the idea). By accepting a contradiction, we forfeit the ability to judge truth and falsity. Thus we disappear within the realm of actuality itself unable to carve up the realm but also unable to be outside of it. It is for these reasons that propositions are always stated within the framework of logic (which is why all propositions can be translated into a logical form) and thus cannot address anything outside of logic. If propositions cannot express something, not only is it technically illogical, it is impossible; we cannot think illogically because it wouldn’t be thinking at all. Thus, there is no space outside of the space of actuality.

We can understand logic’s function in two ways for Wittgenstein: it allows us to make explicit the implicit conclusions contained in a set of propositions and it allows us to reformulate a proposition but due to the lack of sense of logical connectives, we fail to express any new knowledge. Regarding the former, Wittgenstein writes: “A proposition affirms every proposition that follows from it” (1921, 5.124). One may think that through deducing conclusions from sets of propositions one is deriving new knowledge, e.g. deducing that “Felix is a mammal” from “If X is a cat, then X is a mammal” and “Felix is a cat” is ampliative knowledge. This is only an illusion, however, due to our finite intellect. If we had an infinite intellect (such as an infinite being such as God would) all of the implicit propositions contained in the set {“Felix being a cat entails Felix being a mammal”, “Felix is a cat”} would be apparent to us, including “Felix is a mammal”. In regards to reformulating a proposition as a function of logical laws, Wittgenstein writes that “‘logical constants’ are not representatives; that there can be no representatives of the logic of facts” (1921, 4.0312). Wittgenstein doesn’t believe that logical constants express a sense. He gives an example in his text of $\neg p$ and ‘p’ arguing one doesn’t understand $\neg p$, rather one understands another entity, ‘p’. One doesn’t understand the state of affairs as the negation of something; rather we understand the states of affairs as something. Therefore the symbol $\sim$ is
merely formal. We do not understand \( \sim p \) as a complex state of affairs; rather we just understand it as a state of affairs. This analysis extends to all logical connectives, such as horseshoe or wedge. So, laws such as DeMorgan’s laws, which allow us to rewrite the form of a logical proposition, do not impact the sense expressed because the connectives themselves don’t express a sense. For example, \( \sim (p \land q) \) can be rewritten as \( \sim p \lor \sim q \), and because the connectives don’t represent a sense, we haven’t derived new knowledge. We have only changed the form of the proposition.

The practice of logic, unlike an empirical science, does not extend our knowledge. God can lay down the laws of nature because the empirical world is full of content that must be governed. Thus one is able to study the laws of nature and learn *something* about the world (about empirical content). One cannot, however, study the laws of logic because there is no content to study. It is not as if when one learns the notational representation of a logical law they learned something that they did not know before – in fact there isn’t any content to know. Instead they are only making explicit the underlying form of their thoughts. It is not as though no one was rational before logical form was made explicit. We have always been bound by these laws; logic as a discipline just helps make these laws explicit.

Thus the laws of inference (which are included in our laws of logic) contain no knowledge. Wittgenstein writes: “‘Laws of Inference’, which are supposed to justify inferences, as in the works of Frege and Russell, have no sense, and would be superfluous” (1921, 5.132). Laws of inference do not have a sense in so far as they can’t agree or disagree about the possible states of affairs. They neither affirm nor deny any possible state of affairs, they only display an aspect of the logical form of reality. Therefore if one is given the following proposition: “Felix being a cat entails Felix being a mammal and Felix is a cat” and one applies modus ponens, one
would derive that “Felix is a mammal”. Yet, the application of this law of inference hasn’t accomplished anything because “Felix is a mammal” was already contained in the original proposition, it just hadn’t been explicitly stated.

We should also note that logically true propositions (tautologies) are senseless because they express no thought and therefore also cannot tell us anything about the world, demonstrating how the logically true doesn’t extend our knowledge. Wittgenstein writes: “a tautology leaves open to reality the whole- the infinite whole- of logical space: a contradiction fills the whole of logical space leaving no point of it for reality. Thus neither of them can determine reality in any way” (1921, 4.463). Stating a tautology, such as “apples are fruits or apples are not fruits” doesn’t express any meaning (or knowledge). We may experience an illusion of meaning due to our having familiarity with the concepts used (apple and being a fruit), but have failed to picture a state of affairs. In order to do so, one must be making a judgment about the state of affairs that can be judged to be either true or false. By producing a proposition that cannot be measured against the state of affairs to obtain its truth-value, one has failed to say anything of consequence and therefore has failed to say anything meaningful. For example, if a student asks whether or not she should write her essay and her friend responds, “you either write your essay or you don’t write your essay”, the friend hasn’t said anything meaningful. Sure, the student can understand what the friend has said, but she hasn’t actually learned anything about his opinion on the subject- she hasn’t received anything meaningful. In both cases, stating the tautology has failed to make any moves whatsoever because it fails to demarcate the world in any meaningful way.

This analysis holds for more complex logical laws as well. We know the following: “if $p$ then $q$ iff $\sim p \lor q$”. Now take the example of Felix the cat. In virtues of the structure of the
tautology stated, we know that: “if Felix is a cat, then Felix is a mammal iff either Felix is not a cat or Felix is a mammal”. All this tautology has demonstrated is the realm of actuality. We know nothing about what kind of thing Felix is. Instead, we have made explicit the logically necessary possibilities.

What we can take away from the Wittgensteinian account is the following: God doesn’t lay down the laws of logic because essentially they aren’t laws. They don’t govern the world; they express the underlying structure of its possibility. They don’t extend our knowledge or have any content; they are merely formal and elucidate the knowledge we already have access to. God gets to create laws that define the world for us, but logic has nothing to define because there is nothing outside of the realm of actuality. According to this account, there are no alternative laws to choose from.

IV. Cases of Logical Revision:

Now that we have established the basic assumptions that the Wittgensteinian makes about the nature of (our understanding of) the laws of logic, we can rehearse examples of cases that this account of logic fails to capture. The Wittgensteinian is tasked with explaining how developments in the study of logic do not count as revisions in order to maintain that logic is infallible. A revision in our understanding of logic isn’t necessarily a negation of our previous formation of a logical law. Chen Bo characterizes revisions in our understanding of logical laws as follows: “strengthening and weakening” the laws, “incorporating, rescinding, and replacing” laws, and “negating” laws.10 Thus, one could still revise our understanding of logic by modifying a formulation of a law or increasing or decreasing the conditions necessary for the application of

a law (strengthening or weakening). And in so far that they fundamentally change what we have
the capacity to reason deductively about, so too do changes in logical notations constitute a
logical revision. The Wittgensteinian’s inability to explain the nature of these cases demonstrates
a problem for the Wittgensteinian picture of logic- specifically that logic is not a branch of
knowledge.

A. Syllogistic logic and First-Order Logic

Wittgenstein’s characterization of the nature of logic is most compatible with first-order
logic. The Wittgensteinian position struggles to account for any system of logic that differs in
any significant way from first-order logic. For example syllogistic logic – the system of logic
developed by Aristotle – is sufficiently different from first-order logic and therefore isn’t
accounted for in Wittgenstein’s account. Graham Priest shows us that syllogistic logic is
inconsistent with first-order logic, i.e. it validates conclusions that first-order logic is unable to
validate. Due to the use of quantifiers in first-order logic and the fact that the two systems aim to
produce true claims for different ends, the existence of the terms in each system is treated very
differently. Thus the two systems are founded on different sets of assumptions about the
existence of the terms in propositions, revealing the non-compulsory nature of classical logic’s
assumptions.

Take the following example concerning cats and the quality of them being mammals and
them having claws. In syllogistic logic, one can deduce the following:

All cats are mammals.

All cats have claws.

Some mammals have claws.
In first-order logic, one would write:

For all x if x is a cat then x is a mammal.

For all x if x is a cat then x has claws.¹¹

Yet, due to the fact that we have not claimed anything about an instantiation of x (an actual cat), we cannot claim that there exists an x such that x is a mammal and x has claws. Thus we have demonstrated an inconsistency between the two systems of logics.

Aristotle’s understanding of the role that logic plays in our reasoning not only results in the inconsistency between the two systems of logic as demonstrated above, it also demonstrates that despite what the Wittgensteinian believes, the assumptions they take as facts are not universally accepted. The purpose of Aristotelian logic is to express demonstrations of knowledge—specifically syllogistic logic is meant to produce scientific knowledge, which for Aristotle necessarily involves objects. Aristotle writes that the subject of inquiry for logic is “demonstration and the faculty that carries it out demonstrative science”.¹² Unlike Wittgenstein, Aristotle wants his logic to be demonstrative. Thus syllogistic logic is developed in order to produce knowledge. Aristotle writes: “a deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so” (Prior Analytics I.1 24b19-20). From the premises “All cats are mammals” and “All cats have claws”, one can produce the knowledge that “Some mammals have claws” -- something different from what was stated. One may argue that if one drew the conclusions “All cats are mammals and have claws” (similar to what one is able to produce in propositional logic—“For all x, if x is a cat then x is a mammal and x has claws”) one would only be making explicit what was already

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¹¹ This example was heavily influenced by the one given in Priest’s essay, “On Alternative Geometries, Arithmetics, and Logics; A Tribute to Łukasiewicz.”

implicit in the premises. Thus syllogistic logic is able to demonstrate knowledge while classical logic in this case can only make explicit knowledge that we already had.

The ability to demonstrate knowledge in Aristotelian logic as opposed to only elucidate information implicit in our propositions as is the case in propositional logic is in part a result of the act of presupposing existence of the terms in the syllogisms. Our premises for syllogisms are either “primary premises” (premises that are known with such certainty they do not require syllogistic justification) or premises justified by other syllogisms. Primary premises are a result of our sense perceptions. Aristotle writes: “…neither can we possess them [(primary premises)] from birth, nor can they come to be in us if we are without knowledge of them to the extent of having no such developed state at all. Therefore we must possess a capacity of some sort…” *(Posterior Analytics* II.19 99b31-34) and this capacity allows us to turn a collection of sense perceptions into a primary premise (such as seeing objects that are dropped falling to the earth allows us to conclude that all objects when dropped fall towards the earth). So our premises are either a result of perceiving objects that exist or are derived from deducing from other premises. Thus, our scientific knowledge (the knowledge that produces the content in syllogisms) is necessarily object involving. One does not form scientifically valid premises apart from the objects as they exist in the world so Aristotelian logic doesn’t need to involve quantifiers. Aristotle only differentiates between the universal and the particular: the universal being true of all while the particular is only true of some. Neither of these, however, are asserting existence, for one cannot judge objects that do not exist--meaning they are not a concern for the Aristotelian.

For the classical logician, all statements can be vacuously true (i.e. true in virtue of the concepts in relation regardless of whether or not the concepts exist). It is the function of the
quantifier to relate the propositions to objects in the world. Thus, one can assert the existence of members of a set that satisfies the conditions of the proposition. While Aristotle was in search of a way to demonstrate knowledge, the classical logician is concerned with formalizing our understanding of logical laws. As a result of developments in mathematics, people focused on relations, such as the relationship between being continuous and having a derivative. These relations can be thought of independent of any object but the quantifier allows us to relate the relations back to objects. This distinction between relations between concepts and the objects which are instantiation of these concepts is completely distinct from the Aristotelian model where there are only the terms.

There are two quantifiers in classical logic, the existential and the universal, and they enable one to draw different conclusions about the relation of the premises to objects that exist. The existential quantifier (there exists…) asserts that there is at least one object contained in the set that the premise is dealing with. A universal quantifier (for all) however doesn’t assert existence; rather, it asserts that the relation demonstrated in the premise is applicable to the set of all things that satisfy the conditions of the premise. In order for one to go from the universal to the existential, one needs an instantiation of the universal claim, i.e. an object that expresses the relation (such as an actual unicorn that is a horse). It is not the form of “P.1: for all x, if x is S then x is T, P.2: for all x if x is S then x is Q” that allows one to draw the conclusion “C: there exists an x such that x is T and X is Q”, it is the fact that the set X where X is the set of all x such that x is S is not empty that allows one to draw the conclusion.

Perhaps the Wittgensteinian would respond by saying that all we have demonstrated is that syllogistic logic is constrained in so far as it only considers terms that are instantiated but that it isn’t doing anything different. One could not prove something in syllogistic logic that isn’t
reflected in contemporary states of affairs. Yet, this seems to be ignoring the fact that syllogistic logic enables the reasoner to perform different lines of reasoning. It is wrong to view them as “limited” or view syllogistic logic as a less complete classical logic, especially in light of it being able to perform deductions that result in true premises that classical logic isn’t able to conclude. The classical logician who still rejects syllogistic logic as significantly different is unable to recognize that the assumptions founding their approach to logic are not compulsory and as a result to recognize the power of syllogistic reasoning.

B. The Evaluative Argument for the Law of Double Negation Elimination

The Wittgensteinian also takes for granted that our recognition of a law of logic is legitimized by the law’s own self-justification, ignoring that there are indeed laws of logic that are adopted for evaluative reasons. Believing logic to be infallible also entails that there is one true set of logical principles that can never be revised. We can imagine the Wittgensteinian to take the position that this true set of logical principles is that which is proposed by classical logic. In his essay titled “Why Logical Revisabilism Is Wrong,” Yanxi Wei writes: “as a subject, logic is concerned with inference. As to the question of how to study inference, modern logicians have given a complete and systematic method, which is formalization”.13 To these philosophers, we have an exhausted understanding of logic resulting from formalizing our understanding of the laws and thus producing an infallible theory of how one ought to reason. However, this argument of formalization does not answer how we come to determine the necessity of certain laws in classical logic-- especially since we have adopted laws for non-formal reasons.

Despite following a program of what logicians thought to be pure formalization of the laws of logic, some laws required justification that could not be derived from formalization. This is true in the case of the law of double negation elimination (LDNE). The law of double negation elimination states that $p$ can be derived from $\neg\neg p$.\textsuperscript{14} This rule is predicated on assumptions that asserting the negation of a negated term is identical to asserting the term. Thus the adoption of the law rests on it being self-evident. It is clearly the case that LDNE is self-evident in mathematics where one sharply defines concepts – meaning that things are either $p$ or $\neg p$. Thus we are licensed to equate $\neg\neg p$ with $p$.

Notice, however, that LDNE is adopted as a law based off of the judgement that it is useful in regards to mathematical content. Outside of mathematics there are cases where it appears as though we are aren’t licensed to perform LDNE. Take, for example, in everyday language. While being asked whether or not her friend X was a liar, Y responds “X is not not a liar”. Are we licensed to deduce that X is a liar? Not necessarily. Besides the fact that it isn’t clear what Y means, we can argue that even if we knew all of the same information about X that Y knows, we may not want to eliminate the double negation. For example, person X may often omits details about a situation or may answer questions in such a manner that many details are vague. Surely we would not want to claim that X is not a liar, but we may not want to claim that they are a liar- seeing as they are not intentionally deceiving us. Thus from the proposition “X is not not a liar” one cannot deduce that X is a liar.

\textsuperscript{14} This law is similar and in some cases identical in function to the law of excluded middle ($p$ or $\neg p$ or necessarily the case). The law of excluded middle, however, implies that things are either true or false while double negation elimination only makes claims about terms determined to have a truth-value. For these reasons I will not take up the case of the law of excluded middle.
The Wittgensteinian may respond by saying that this is a question of pragmatics, not of semantics, and thus doesn’t concern the laws of logic. In saying X is not not a liar, Y is manipulating context in order not to commit herself to judging the state of X’s character. There is, however, an answer to whether or not X is a liar, thus not not being a liar implies being a liar. Regardless of whether or not that is a correct assessment of the situation, this example still demonstrates that the laws of classical logic, which are meant to bind our understanding universally, do not bind our deductive reasoning in cases of everyday language. Further, I do believe that one could ignore pragmatics and focus just on semantics and still not be able to apply LDNE to this case. In everyday language we are often reasoning about non-sharply defined concepts (unlike in mathematics), such as in the example of some person being bald. If someone is balding, what do we want to say about whether or not they are bald? Perhaps all we would want to say is that they are not not bald.

As Hartry Field argues, the uncertainty about whether or not a law is self-evident is a result of logic being in part “evaluative”. Field presents a distinction between two types of a prioricity: a strong one and a weak one. Most people claim that our understanding of the laws of logic are strong a priori –meaning that they are never vulnerable to revision. I, however, am claiming that they are weak a priori-- meaning that while our formulations of the laws of logic are not affected by empirical evidence, they are not infallible. It is the evaluative component of our understanding of the laws of logic that allows for this type of fallibility. Field writes: “what is at issue rather is evaluation: the evaluation of standards that (i) allows for the acceptance of certain rules or principles without empirical evidence and that (ii) make no allowance for the

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15 I was referred to this counter point by Joel Yurdin.
retraction of those rules or principles on empirical grounds” (1998, 11). Thus the self-evident character of our current laws of logic is derived from evaluation. The question of whether or not to add LDNE to our set of logical laws depends on whether or not we judge it (evaluate it) to be useful to our exercise of reason. In the case of contemporary mathematics, we judge it to be necessarily true, while in the case of everyday language, we judge it not to be a useful law.

Not only do we have now have cases where the laws that bind our reasoning vary depending on the content being reasoned about, we also have included a fallible component to formulating our laws of logic-- evaluation. We are all capable of evaluating the importance of a law incorrectly, which would force us to revise our understanding of the laws of logic. We can also imagine a case where our values or needs change which will prompt us to re-evaluate the laws under different circumstances thus allowing for the possibility to revise the laws that we understand to be logical laws. Accepting or rejecting LDNE is an indication both of our current values as well as indicative that previous logical systems were inadequate and were in need of revision in order to characterize how one is reasoning. A logic that includes LDNE in its set of laws is inconsistent with a logic that doesn’t include LDNE.17 The Wittgensteinian is then forced to choose one system to be correct. Yet, we have just demonstrated that in choosing one system, we limit ourselves from properly characterizing our reasoning in all disciplines. We ought to then abandon the Wittgensteinian commitment to only one true logic in order to better encapsulate all types of deductive reasoning.

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17 As evidence that LDNE transforms logical systems, LDNE is not a valid move in intuitionistic logic and the set of rules of intuitionistic logic is different than that of classical logic.
C. Notation as Mediating our Access to Truth

The final example I will use to put pressure on the competency of the Wittgensteinian account is the role that notation plays in mediating our formulation of logical laws. Wittgenstein believes the world to be the “totality of facts, not of things” (1921, 1.1). Rather than the world being constituted by objects, the world is constituted by facts which are expressed by propositions. Thus there is no independent “object” that confronts what we understand to be the laws of logic in order to assess whether or not these laws preserve truth-value. For the Wittgensteinian, the fact that different systems of notation can all be translated into one another demonstrates that there are no significant differences between the logical notations that would affect the possible truths produced in the logic. Just as we demonstrated in our first case, one can manipulate propositions expressed in syllogistic logic in order to express them in first-order logic, such as “All cats are mammals” becomes “For all x, if x is a cat then x is a mammal”. The Wittgensteinian would argue that the two propositions may have cosmetic differences, but that the different notations preserve the truth-values. This ignores, however, that different notations enable us to represent concepts differently and that this difference mediates our access to truth. We have already demonstrated how classical logic’s quantifier notation allows it to assert existence rather than presuppose it, unlike Aristotelian logic. Thus, we must account for logical notation as a mediator, which puts pressure on the Wittgensteinian position that all logical notations must function identically.

In order to demonstrate how notation typically affects our ability to represent knowledge, we shall turn to mathematics and compare Euclidean Geometry to algebraic reasoning. Take, for example, a circle. Euclidean diagrams are concerned with the geometric figures (which are represented two-dimensionally) while the algebraist is concerned with the definition of a circle,
\[ x^2 + y^2 = r^2 \] (which expresses the relations between different variables).  

We already see how the two notations represent the content completely differently – one represents a two-dimensional figure while the other is concerned only with relations. In regards to how different mathematical notation have different powers, Danielle Macbeth writes that Réné Descartes, who successfully realizes the power of algebraic notation for mathematical reasoning, “came to read the symbolism of algebra… as a fully meaningful language within which to reason through a metamorphosis in the way he regarded drawn Euclidean figures”.  

The algebraist, such as Descartes, looks at Euclidean figures in a completely different way from the Euclidean. It isn’t merely a translation between two different mathematical notations where the difference between the two is only the aesthetics of their notation, it’s a “metamorphosis”, a transformation in the way we access the concepts represented. This also enables us to perform mathematical tasks previously impossible in Euclidean Geometry. For example, one is able to use a Cartesian plane and not only describe shapes, lines, and points, one can also describe functions.

As demonstrated by the mathematical example, different notations highlight different aspects of content. Just as Euclidean geometry represents geometrical figures, algebraic reasoning represents relations between variables. Many proofs can be translated from one notation to another – such as the demonstration for the Pythagorean Theorem – and the focus on different features thus changes what knowledge we are accessing. In Euclidean geometry, one actually sees how the theorem applies to two-dimensional shapes while in algebraic notation, one initially understands the theorem in terms of numbers and their relation to one another.

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18 If using Cartesian coordinates, one writes: \( (x-h)^2 + (y-k)^2 = r^2 \).

By analyzing the “notation”\textsuperscript{20} used in syllogistic logic as opposed to the notation used in classical logic, we can see how these two notations present concepts to us quite differently and therefore alter the knowledge (the truth) that we are able to access. Let us reexamine the earlier example of cats being mammals. In the syllogistic proof, we demonstrated scientific knowledge about terms. When we transitioned into classical logic, our focus shifted to making explicit the necessary relations between concepts. Although we weren’t able to conclude that there existed a clawed mammal, we can produce the following according to the laws of classical logic:

\begin{align*}
\text{For all } x \text{ if } x \text{ is a cat then } x \text{ is a mammal.} \\
\text{For all } x \text{ if } x \text{ is a cat then } x \text{ has claws.} \\
\therefore \text{ For all } x, \text{ if } x \text{ is a cat, then } x \text{ is a mammal and } x \text{ has claws.}
\end{align*}

Unlike how I used this example above, I am not trying to claim that syllogistic logic doesn’t also produce this knowledge. Although the conclusion doesn’t fit the format of the syllogistic proof (one doesn’t formally write that “All A is B” and “All A is C” so “All A is B and C”), surely the Aristotelian would agree that syllogistic reasoning justifies our belief that “All cats are mammals and have claws.” Rather, I’m examining how the derivation of this conclusion demonstrates that classical logic’s notation highlights different aspects of the logic. In the above proof, we haven’t produced any new knowledge, but we brought attention to the relationship between being a cat, being a mammal, and having claws. More specifically, we have focused on the relationship between these concepts independent of the figures that embody them. While in syllogistic logic we are always concerned with the objects that embody the terms, in classical logic the nature of the universal quantifier allows us to separate the two and only worry about the concepts.

\textsuperscript{20} Syllogistic logic doesn’t necessitate the use of notation as does classical logic. Yet, it does require propositions to be put into a certain form and uses particular words to represent certain ideals (such as “some” to designate at least one”) and in those ways appears to functions as a notation.
themselves. In this way, the two logical systems enable us to focus on two different approaches to knowledge.

The Wittgensteinian position doesn’t wholly account for the variation in what comes across as salient in logical proofs as a result of the notation used. As we have demonstrated, notations in many ways acts a mediator between us and the truth that they are mediating. As a result of these different types of mediation, the truth we arrive at varies. What does this entail about the nature of logical laws? As Penelope Maddy claims, logical notation requires digestion in order to take our intuitive logical notions and represent them linguistically:

“The fundamental modes of conceptualization do provide us with rudimentary versions of the logical primitives, which mean that the logical laws governing those primitives will be correct for our thought and that they will seem obvious to us. But several linguistic extensions and at least two substantial idealizations are needed before we can reach anything like the laws of logic.”

Although we have intuitions about the ways in which our reasoning is bound, our formulations of these laws are necessarily linguistic and thus our intuitions undergo “substantial idealizations.” This means that different linguistic representations possibly idealize our intuitions in different ways. As the demands on our reasoning change, so does how we idealize these intuitions. As mentioned earlier, classical logic is in part a response to developments in mathematics. Previous formulations of logical laws didn’t allow for one to abstract the concepts from the objects that embodied them. Yet, classical logic notation does just that by introducing the quantifier and allowing one to represent concepts as variables (using “x” as the subject as opposed to a noun such as “cat”).

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V. Being True and the Realm of Actuality

The three cases presented in section IV demonstrate how the Wittgensteinian conception of what we understand to be the laws of logic is unable to wholly capture the nature of logic. What characteristics of logic then is the Wittgensteinian ignoring? I will argue that by acknowledging two characteristics of logic – that logic is the study of being true and that what we perceive to be the realm of actuality evolves – we can move towards a more complete picture of logic as a branch of knowledge as opposed to our formulation of logical laws as being merely formal.

In his introduction to Grundgesetze, Frege distinguishes between being true and being held as true. Frege writes: “I understand by logical laws not psychological laws of holding as true, but laws of being true”.22 Studying being held as true involves studying psychology meaning that in learning what we understand to be the laws of logic would only produce what is humans consider to be true. Frege, however, is concerned with the laws of being true – meaning that being true is being in some way independent of the human psyche.

At first blush, one might imagine the Wittgensteinian not taking issue with what Frege has said. The Wittgensteinian doesn’t advocate for a picture of logic that is reducible to human psychology. If this were the case, then there would be no reason to argue that there is no space outside the realm of actuality because any rational being with a different logic would technically be operating outside the realm of actuality. For the Wittgensteinian, however, there is no being true in the case of logical truths. Take for example the law of identity. The law of identity doesn’t express anything about the nature of being true because it doesn’t express anything

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whatsoever. It’s merely formal. Frege does believe, however, that due to logic being concerned with *being true*, we could imagine non-human rational beings who reject what we understand to be the laws of logic. These logical aliens could, for example, reject our law of identity, i.e. being *p* doesn’t entail being *p*. Frege responds to this problem as follows:

“…this impossibility of our rejecting the law does not prevent us from supposing that there are beings who do reject it; but it does prevent us from supposing that these beings are right in doing so; it also prevents us from doubting whether we or they are right. At least this goes for me.” (1893, 204).

Frege reinforces that what we typically refer to as “the laws of logic” are actually what we understand to be the laws of logic. When confronted with a non-human rational being who recognizes a different set of logical laws, we are not able to determine who is correct. Studying *being true* entails studying something (the way it is to for something to be true) but as finite beings we are constantly vulnerable to only achieving knowledge of what we *hold to be true*. For this reason, we can never know whether we or the other rational being uses the correct rule.

Frege writes: “…logic can only answer by reducing it to another logical law. Where that is not possible, logic can give no answer” (1893, 204). This is similar to the claim made by Field: part of accepting or rejecting formulations of the laws of logic involves non-logical principles – especially in the cases involving what we understand to be fundamental logical laws (fundamental meaning laws that are accepted due to their self-evidence and are not derived from other, pre-existing laws) such as the law of identity. We can’t demonstrate whether or not they’re true because the very tools we use to derive truth are the laws themselves. We can’t, however, dispose of them either unless we are willing to forfeit our ability to think. All we can do is
continue to strive for a logical notation better at mediating our access to truth while acknowledging we are always vulnerable to merely expressing what we hold to be true.

The second point I believe will be useful in order to construct a better characterization of logic is to consider that what we perceive to be the realm of actuality also changes. Studying *being true* also creates space for our revising what we understand to be logical laws. But what prompts these changes? What caused post-Cartesians to alter their conception of logic? What we are able to comprehend, what is considered to be possible, evolves through conceptual revolutions. This isn’t claiming that what is actually possible changes or that the underlying logical structure that defines possibility changes. As Graham Priest writes: “an applied logic is a theory of logical relations. In particular, then, when we change our logical theory, it does not follow that what principles of logic are valid also changes, any more than when we change our theory of geometry, the geometry of the cosmos changes” (2003, 462). The laws themselves aren’t being revised – just how we understand them. This theory compliments Frege’s example of logical aliens. In a sense, we can imagine ourselves someday filling the role of a logical alien with a completely different formulation of logical laws.

Our failure to recognize that what we perceive as the realm of actuality cuts us off from accessing a richer understanding of previous logical systems. As a result of us always approaching previous logical systems with a contemporary lens (as classical logicians at this moment), we are unable to assess to what degree logical systems vary. In her book *Realizing Reason*, Macbeth writes: “our project of understanding how new shapes of consciousness can emerge in the course of history cannot succeed if elements of consciousness that are essentially late, that can emerge only out of some earlier shape of consciousness, are then read back into that earlier chapter” (2014, 12-13). As I have argued, the Wittgensteinian has done just that: they
have taken our contemporary logical system as absolute and have done violence to previous
logical systems and aspects of logical notation by trying to understand it as a form of classical
logic. Philosophers often confuse the words “all” and “some” in syllogistic logic as standing in
for early quantifiers. Yet, as I have demonstrated, “all” and “some” in Aristotle play
fundamentally different roles than quantifiers. The only way to understand how Aristotelian
logic is meant to demonstrate knowledge according to Aristotle (extend our knowledge as
opposed to classical logic which can only make explicit the implicit) is to abandon the
assumptions made by classical logicians that we have forgotten are assumptions. Only when we
study and understand the motivation behind syllogistic logic, what types of things constituted
Aristotelian knowledge, and other aspects of ancient culture can we get an accurate
characterization of syllogistic logic. Without understanding what Aristotelians thought their
realm of actuality to be, we cannot completely understand their logical system.

It is also the case that when we understand the possibility that what we conceive as the
realm of actuality can change, we also then are fully able to appreciate how logical revision takes
place. This view is especially relevant among philosophers such as Hilary Putnam who argue
that due to developments such as Quantum physics, we should revise the laws of logic.²³
However, the only thing prompting this revision is empirical evidence. The revision would force
us to reject a logical principle that still appears to us as self-evident. Yet, Field writes in regards
Quantum revisability that:

“There are obstacles to explaining what the inductive methods that would go with such a
logic would be like. (I'm now speaking only of the conceptual obstacles to explaining

²³ To clarify- in quantum mechanics, it appears as though one cannot preserve the certain truth conditions under
current formulations of logical laws. Specifically, the following two propositions don’t have the same truth-values:
$p \land (r \lor q)$ and $(p \land r) \lor (p \land q)$. In classical logic, these two propositions are equivalent and thus always
produce the same truth-value as the other according to the law of distribution.
what it would be like to use a nondistributive logic; not of the still further conceptual obstacles to explaining what it would be to use an inductive method that would allow the shift from classical logic to nondistributive logic on empirical grounds).” (1998, 16)

As Field points out, we cannot actually revise our current system until what determines the laws of logic too is altered- i.e. the realm of actuality is altered. Until we overcome this conceptual obstacle, we cannot revise what we currently understand to be logical laws to accommodate the content of quantum physics. Thus it is the change in what we perceive to be the realm of actuality (in this case the overcoming of conceptual obstacles) that allows one to revise their logical principles.

VI. Conclusion

The assumptions about what we understand to be the laws of logic that the Wittgensteinian take to be compulsory are insufficient for properly characterizing logic. Due to many people implicitly accepting Wittgenstein’s assumptions, such as the infallibility of logical laws and lack of content expressed in logic, we have maintained this outdated image of what logic does/is capable of doing. By recognizing that these assumptions are not compulsory, we will be able to characterize previous logical systems and open ourselves up to reformulating new systems of logic in order to accommodate the current demands of our intellectual practices.

Furthermore, if we continue to accept the Wittgensteinian position as the compulsory position, we will not be to understand how we reason deductively. Wittgenstein believes that we already have complete knowledge of how to reason deductively as captured by classical logic. This highly mechanistic style of reasoning however ignores many aspects of how we actually
reason deductively as I have examined in this paper. If we ever want to capture human reason, we must abandon this mechanistic model and conceptualize logic as a branch of knowledge.
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