

# Measurement Theories in Quantum Mechanics

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# Chapter 1

## Introduction

### 1.1 Intent

The purpose of this document is to provide a mildly technical introduction to the unique character of quantum physics as they pertain to the problems of measurement, and some proposed solutions. I conclude with a discussion of fundamental and broadly valid restrictions on quantum theory.

### 1.2 Overview

All fundamental problems with quantum mechanics come from measurement. The linear time evolution of Quantum Mechanics (QM), described by the Schrödinger equation, is completely trustworthy; it describes all verified processes correctly. But linear time evolution seems to be interrupted by almost every sort of interaction with a quantum system. Stray particles and incoherent light introduce irregularities that appear (upon first inspection) to disrupt linear time evolution. But physicists believe that all light and matter obey quantum mechanics, so the quantum linear time evolution rule ought to hold in all cases.

However, the behavior of equipment in laboratories seems at odds with linear time evolution. When quantum mechanics was being formulated, an *ad hoc* proposal was introduced to describe the action of a measurement apparatus<sup>1</sup>. This proposal, often called the “collapse postulate,” states that a measurement apparatus corresponds to a quantum-mechanical operator, and after a quantum-mechanical system interacts with a measurement apparatus, the quantum system is left in an eigenstate of the corresponding measurement operator and the measurement apparatus indicates the eigenvalue corresponding to that eigenstate. The collapse postulate also specifies some probability for the system to be left in each eigenstate; the probability depends on the quantum system’s state before measurement.

Unfortunately this alleged behavior of laboratory equipment is incompatible with the rules of quantum mechanics. I shall examine four attempts to reconcile this behavior with quantum mechanics. The first, called a Spontaneous Collapse theory, changes the rules of quantum mechanics. The second, a Hidden Variables theory, adds some additional rules. The third, the Many Worlds Interpretation, alleges that the quantum rules already explain essentially everything. The fourth, Decoherence, makes a substantially similar allegation, but comes to different conclusions.

Decoherence, the most widely accepted and theoretically advanced of these attempts, explains how linear time evolution leads to a phenomenon which appears strongly nonlinear; [Sch04] calls this a “byproduct” of the Decoherence Program, but I see it as a primary motivating factor. Many fundamental difficulties with QM come from the fact that the classical technique of studying isolated systems becomes problematical in the new physics [Boc05a]. These difficulties turn our

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<sup>1</sup>A piece of equipment or human usually consisting of many atoms.

attention to past theoretical developments.

### 1.3 New Theories from Old

David Mermin asks, “What is Quantum Mechanics trying to tell us?”. He answers this by analogy to the answer to an earlier question, “What are Maxwell’s Equations trying to tell us?” To this earlier question, we answer, “Fields in space have physical reality; the medium in which they propagate does not.” Mermin answers his question, “Correlations have physical reality. That which they correlate does not.” I shall not discuss Mermin’s answer further; such truly fundamental questions are beyond my ability to analyze. Quantum Mechanics has simpler and more obvious things to say to us, just as Maxwell’s equations, having been proposed to explain electricity and magnetism, also describe light. Just as Relativity offers a deeper understanding of electromagnetic phenomena, Mermin’s correlations may offer some new structure to quantum mechanics, but here I seek only to analyze some aspects of measurement.

What, then, is QM trying to tell us? With its Hilbert space structure and associated operators, we can describe an elementary system, the spin of nuclei in a magnetic field. For simplicity, consider spin- $\frac{1}{2}$  nuclei such as hydrogen. Classically, the nuclei would be distributed in energy states according to their Boltzmann factors. Quantum mechanically, we can assume this also. If a tuned radio-frequency pulse is applied at resonance (approximately 15 MHz), the spins will change state. After a pulse of most lengths, an oscillating magnetization can be detected as the nuclear spins, now tilted to some angle relative to the magnetic field, precess about it. The magnetization will decay back to the equilibrium value as the spins return to thermal equilibrium.

This description captures much of the physics involved in performing Nuclear

Magnetic Resonance experiments, but there is a detail I have neglected. At the beginning of the experiment, why are the nuclei in a mixture of energy eigenstates? Classically, the nuclei were assumed to be distributed among the available states so as to maximize entropy, but only states of definite energy were available. The answer is that the nuclei are not simply in a mixture of energy eigenstates, but rather distributed among the available states so as to maximize entropy. These distributions are here equivalent. Often, as in this case, a simple assumption is true for a deeper reason. Sometimes a simple assumption is only approximately true.

This seems to be the case with measurement. The orthodox theory is simple and only approximately true. To improve on it, one wishes to determine what quantum mechanics can say about a system. In determining what quantum mechanics can say about a system, it helps to consider a trivial system. Consider a flask of water in a magnetic field. What can one say about the proton spins of the hydrogen in the water? An aside: what are the limits on what one can say about a system? In an absolute sense, there are none. A layperson say whatever he pleases about proton spins, even “Protons enjoy camping, walking on moonlit beaches, and waking up early to make coffee.” Professional ethics bind engineers. Engineering societies provide codes of conduct, and so an engineer might only say what she can stand behind. She might say, “Proton spins align with an external magnetic field approximately linearly in the low field limit; many materials with nuclear or electronic unpaired spins follow Curie’s law.” A physicist has fewer restrictions, but it’s worth trying to make them explicit. I attempt to enumerate mine, based on the thoughts of Carlo Rovelli [Rov04] and Gennaro Auletta [Aul01].

- Good physical explanations are economical; they explain a wide range of

phenomena with a minimal set of principles.

- New theories often come from old; uneducated radicalism is often sterile.
- Still, examine closely and critically the assumptions and statements of current theories. Kepler, Einstein, Ptolemy, and Planck all worked diligently on contemporary theories before making their great advances.
- Don't hesitate to make new predictions, but they must be testable.



# Chapter 2

## A Brief Review of Mechanics

“It is important to understand which parts of Newton’s laws are based on experiment and which parts are matters of definition. . . . We start by appealing directly to experiment. Unfortunately, experiments in mechanics are among the hardest in physics because motion in our everyday surroundings is complicated by forces such as gravity and friction.” [KK73]

The next two sections cover Classical Mechanics (CM). CM is a way of modeling the world as a collection of bodies and forces. Each body has a definite position and momentum; forces, determined by the current and past positions of the bodies, act by inducing accelerations in the bodies.

The last few sections briefly discuss Quantum Mechanics, which models the world as a set of vectors in a Hilbert space with time evolution determined by operations on these vectors.

Despite these deep differences in mathematical structure, the theories share many fundamental physical concepts; energy, entropy, action, and microscopic reversibility are central to both theories.

## 2.1 Newtonian Mechanics: $\vec{F} = m\vec{a}$

Historically, Newtonian Mechanics, much like Quantum Mechanics (QM), began with a vaguely defined set of concepts. Despite early debates over the relative importance and meaning of concepts in the theory, the Newtonian way of thinking led to great theoretical and experimental progress in physics over the two hundred years that passed before the work of Mach and others provided rigorous understanding of the meaning of Newton's definitions and assertions. A brief sketch of the classical theory is now necessary to understand those features of the quantum theory that bother some scientists.

### 2.1.1 Newton's Laws of Motion

Although Newton believed in absolute time and space, belief in his laws does not require these beliefs. Newton's laws describe certain characteristics of bodies and their motions and interactions in space. We can, from these laws and experiment, infer practically applicable ideas of what a "body" or a "force" is.

**Newton's First Law.** *Bodies continue at a uniform velocity unless acted upon by an unbalanced force.*

Kleppner & Kolenkow state this law as "Inertial frames exist." An inertial frame is one in which objects not experiencing unbalanced forces exhibit constant velocity motion. That many inertial frames exist is experimental fact.

**Newton's Second Law.** *Given a body of mass  $m$  experiencing forces  $\vec{F}_i$ , its acceleration is given by  $\sum_i \vec{F}_i = m\vec{a}$ .*

From this law, we can infer relationships between force, mass, and motion. Given some procedure for generating a given force, if we apply this force to two

bodies experiencing no other forces, the ratio of their accelerations will be the ratio of their masses. If two procedures cause the same body to undergo the same acceleration, the two procedures must generate the same force. In addition, forces add linearly to produce summed accelerations.

**Newton's Third Law.** *Often stated as "Every action has an equal and opposite reaction." The weak form of the third law states that forces appear in pairs of equal magnitude and opposite direction; the strong form additionally states that pairs of forces lie along the line connecting the points on which they act.*

### 2.1.2 Mathematics and the Connection to Experiment

I have stated Newton's second law in the modern notation of vector analysis, but none of the laws above mention what kind of mathematical entities describe bodies, forces, or the motion of bodies. All numbers above are real. Mass is a real scalar, forces and accelerations are triples of real numbers.

Part of the power of Newton's laws is the framework for understanding they provide, but part is their ability to describe experiments and guide design of physical apparatus. As it is in the connection to experiment that QM differs most significantly from earlier physics, I shall strive for clarity in explaining the interpretation of experiments in Newtonian Mechanics.

Newton's laws provide for dynamical systems to evolve independent of human interaction. Appropriately expanded with definitions of forces in terms of particle positions (and, in the case of electromagnetism, past positions), these laws are deterministic. If one believes these laws, then one will expect measuring devices to follow the same dynamics as the system being investigated, so let us try to follow what their reasoning might be. Given the ability to generate and detect test particles of arbitrarily small mass (and such devices do not contradict the Newto-

nian dynamical laws), the position and velocity of bodies can be measured with arbitrarily small disturbance.

**Simple Harmonic Oscillator Example.** *Consider a mass attached to a spring:*

$$\vec{F} = -kx\vec{t} \quad (2.1)$$

$$\vec{F} = m\vec{a} \quad (2.2)$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \quad (2.3)$$

This familiar system exhibits sinusoidal oscillations centered around zero displacement under most initial conditions. If the position of the oscillating mass is determined by, for instance, launching a particle from some point on the  $x$ -axis at the mass and recording the time it takes to return, the disturbance of motion of the oscillating mass will be minimal, provided the velocity and mass of the particle are small. This will stand in sharp contrast to the quantum situation.

## 2.2 Hamiltonian and Lagrangian Mechanics

The mechanics of Hamilton and Lagrange neither conflict with nor expand Newton's mechanics, but their viewpoint and associated mathematical techniques provide a clearer view of aspects of the physical world. As Quantum Mechanics almost universally relies on the Hamiltonian, I shall introduce it briefly.

### 2.2.1 Lagrangian Mechanics

Although the Lagrangian can be derived directly from Newton's laws of motion, it is more enlightening to develop it from Hamilton's principle and the Calculus

of Variations.

**Definition of Kinetic Energy.** *The kinetic energy  $T$  is the energy required to accelerate a free particle or system of noninteracting particles to velocity  $v$  or velocities  $v_i$ . For a single particle,  $T = \frac{1}{2}mv^2$ . For a system of particles,  $T = \sum_i \frac{1}{2}m_i\vec{v}_i \cdot \vec{v}_i$ .*

**Definition of Lagrangian.** *The Lagrangian  $L$  is defined as the difference of the kinetic and potential energies:  $L = T - V$ .*

**Definition of Action.** *The action  $S$  is the time integral of the Lagrangian as the dynamical system moves along some path in phase space.*

$$S = \int L dt \quad (2.4)$$

**Hamilton's Principle.** *A dynamical system follows the path whose action has a stationary value. This can be written  $\delta S = 0$ .*

**Euler-Lagrange Condition.** *Given a function  $L$ , we seek to determine  $q_i(t)$ ,  $\dot{q}_i(t)$  such that  $S = \int_a^b L(q_i(t), \dot{q}_i(t), t) dt$  has a stationary value. A fundamental result in the calculus of variations is that for  $\delta S = 0$ , the **Euler-Lagrange condition***

$$\frac{\partial}{\partial q_i} L - \frac{d}{dt} \left( \frac{\partial}{\partial \dot{q}_i} L \right) = 0 \quad (2.5)$$

*must hold.*

For problems in Lagrangian Mechanics, the Lagrangian can usually be written down by inspecting the physical system to be analyzed; the Euler-Lagrange condition then allows the equations of motion to be determined.

**Simple Harmonic Oscillator Example with Lagrangian.**

$$T = \frac{1}{2}m \left( \frac{dx}{dt} \right)^2 \quad (2.6)$$

$$V = \frac{1}{2}kx^2 \quad (2.7)$$

$$L = \frac{1}{2} \left( m \left( \frac{dx}{dt} \right)^2 - kx^2 \right) \quad (2.8)$$

$$-kx - \frac{d}{dt} \left( m \frac{dx}{dt} \right) = 0 \quad \text{Euler-Lagrange Condition} \quad (2.9)$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \quad (2.10)$$

*Note that Equation 2.10 matches 2.3 above, as expected.*

**2.2.2 Hamiltonian Mechanics**

The Euler-Lagrange equations are a set of second-order differential equations. They can be transformed into twice as many first-order differential equations, and the usual way of finding them is through the Hamiltonian.

**Definition of Hamiltonian.** *The Hamiltonian is the Legendre transform of the Lagrangian,*

$$H(q_i(t), p_i(t), t) = \sum_k \dot{q}_k(t)p_k(t) - L(q_i(t), \dot{q}_i(t), t) \quad (2.11)$$

*For some systems,  $H = T + V$ .*

The partial derivatives of the Hamiltonian with respect to the positions and momenta yield first-order differential equations for the motion.

**Canonical Equations of Motion.**

$$\frac{\partial H}{\partial q_i} = -\dot{p}_i \qquad \frac{\partial H}{\partial p_i} = \dot{q}_i \qquad (2.12)$$

These first-order differential equations can be solved for the future behavior of the system. Compare the mathematical complexity of the following example to example 2.2.1 above; for this simple system, the two methods are equally complex.

**Simple Harmonic Oscillator Example with Hamiltonian.**

$$H = \frac{1}{2} \left( m \left( \frac{dx}{dt} \right)^2 + kx^2 \right) \qquad (2.13)$$

$$\frac{\partial H}{\partial x} = -\frac{d}{dt} m \frac{dx}{dt} \qquad (2.14)$$

$$kx = -m \frac{d^2x}{dt^2} \qquad (2.15)$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad (2.16)$$

*Note that Equation 2.16 matches 2.3 and 2.10 above, as expected.*

**2.2.3 Conclusions**

The Hamiltonian and Lagrangian formulations of Newtonian Mechanics are very similar in use; both yield differential equations that can be solved for variables of motion. In Quantum Mechanics, the situation is much the same: the Hamiltonian and path-integral (based on the Lagrangian) formulations of QM yield differential equations that can be solved for the future state of the system.

Remember that in all three mathematical representations of Classical Mechanics, a small body can be described by seven numbers: a real scalar for mass, a real 3-vector for position, and a real 3-vector for velocity. QM will be different.

## 2.3 The Hamiltonian in Quantum Mechanics

Classically, under certain commonly-applicable conditions, the Hamiltonian for a system is also the total energy. The same is true in QM. However, in Quantum Mechanics systems do not always have definite total energy. Quantum mechanically, the energy of a system is represented by an operator, the Hamiltonian. Only eigenvectors of this operator have definite energy; superpositions of them do not. In general the state of a system is not an eigenvector of the Hamiltonian, so in general a quantum mechanical system does not have a definite energy.

In Quantum Mechanics, the set of  $q_i(t)$  and  $\dot{q}_i(t)$  is replaced by a set of entities  $|\psi_i(t)\rangle$ . From these  $|\psi_i\rangle$ , position and velocity can be estimated but are not always certain. Just as Equation 2.12 determines the time evolution of a classical system, the Schrödinger equation,

$$H|\psi\rangle = i\hbar \frac{\partial}{\partial t} |\psi\rangle, \quad (2.17)$$

determines the time evolution of a quantum system.

## 2.4 The Lagrangian in Quantum Mechanics

Classically, it is often stated that a system follows the path of least action. More precisely, Hamilton's principle, that a system follows the path of stationary action, holds.

Quantum Mechanically, systems do not in general follow a single path (or reach a single destination). Instead a probability amplitude can be determined for each possible final state of a system by summing over all paths (sets of  $q(t)$ ,  $\dot{q}(t)$ ):

**Definition of Path Integral.** Call a function  $q(t)$  which has  $q(t_f) = \vec{x}_f$  and  $q(t_i) = \vec{x}_i$

a path from  $t_i$  to  $t_f$ . Then, defining the action for a path as the usual integral of the Lagrangian,

$$S_{path} = \int_{t_i}^{t_f} L(\mathbf{q}(t), \dot{\mathbf{q}}(t), t), \quad (2.18)$$

$$\Phi(\vec{\mathbf{x}}_f, t_f) = \sum_{paths} e^{iS_{path}/\hbar}, \quad (2.19)$$

The amplitude at a point in space and time is the sum over all paths of Equation 2.19. The amplitudes must be normalized to produce unit probability that the system is somewhere, but I have omitted the necessary constant.

## 2.5 Conclusion

The mathematical groundwork laid, in the next chapter I shall discuss the nature of measurement in QM.



# Chapter 3

## The Measurement Problem

### 3.1 Introduction

Quantum Mechanics, the physical science that describes the nature of reality on small scales, has shown unrivaled success at predicting and explaining experimental results since its conception in the early 20<sup>th</sup> century. Despite this success, bitter conflicts over the qualitative physical content of the new theory have been fought since then. The views of Quantum Mechanics (QM) held by the various sides in these conflicts are called interpretations. Most importantly, these interpretations all agree on the application of Quantum Mechanical (QM) principles to experiments; they do not differ when it comes to (even state-of-the-art) chemistry, engineering, and applied physics. However, the value of physical theories does not lie only in explaining what has already been observed and cataloged. Physics is a human endeavor, and humans understand good physical theories; such theories show unexpectedly broad validity. I hope that a better understanding of Quantum Mechanics will open up new avenues of theoretical and applied research.

## 3.2 Quantum Mysteries

### 3.2.1 Probability Rules

The conflicting interpretations of Quantum Mechanics stem from one source: the differing probability rules used by Classical and Quantum Mechanics. The quantum probability rule for measuring a series of two events occurring differs depending on whether or not a measuring device interacts with the system at times between the events; in the former case, it is the same as the classical rule, but in the latter case, it is not.

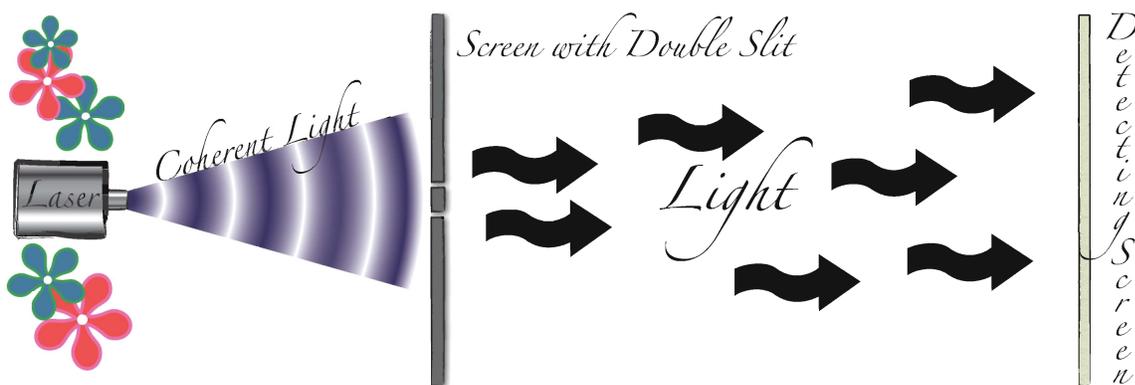
$$\text{Classical: } p_{ac} = \sum_b p_{ab}p_{bc} \quad \text{Quantum: } p_{ac} = \left( \sum_b \phi_{ab}\phi_{bc}^* \right)^2$$

This change in the probability rule gives rise to many counterintuitive statements about Quantum Mechanics. Whereas before the advent of QM, all physical objects followed the same probability rules, regardless of size or complexity, Quantum Mechanics seemed to require that the world be split into microscopic (quantum) and macroscopic (classical) sections. People believed this was necessary because large objects were seen to follow the left, classical rule, whereas QM (which agreed with experiments on small objects) predicted the right, quantum rule. This seemed paradoxical because large objects are believed to be built out of smaller objects; in fact, for large (anything from a speck of dust to a galaxy) objects, the rule on the right averages out to almost equal the rule on the left. The process that causes this averaging is called decoherence, and it gives a physical explanation for the fact that large objects following the quantum rule appear to also follow the classical rule.

### 3.2.2 Probability in Experiments: Diffraction

The noted physicist R. P. Feynman<sup>1</sup> called this elegant experiment “the central mystery of quantum mechanics—the one to which all others could ultimately be reduced.” It shows several interesting properties of the objects many believe to be the fundamental building blocks of our reality. It leads one to believe that these objects come in small, discrete, identical pieces. It also leads one to believe that these objects are sensitive to their surroundings over comparatively large distances. For many years, no one could satisfactorily explain how these properties are compatible.

#### Experimental Apparatus for Diffraction Experiments



#### The Single-Slit Experiment — Necessary Groundwork

The first conclusion one draws from the double-slit experiment, that fundamental objects come in small, discrete pieces, is seen also in a simpler experiment. Consider only the laser and the detecting screen from the above figure. If the laser is turned on at low intensity and a camera is pointed at the screen, pinpoints of light will flash one at a time at various points on the screen. These pinpoints of light will all be of the same brightness and size. This has led many physicists to infer

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<sup>1</sup>[Gle92]

that the laser produces localized and quantized packets of energy called photons.

Consider now the arrangement in the above figure, but with only one of the two slits open. The pinpoints of light will now be distributed over the screen as in the first or second column of figures, depending on whether the top or bottom slit is open. Most of the light hits the screen directly through the slits, although some hits the screen above or below them. Always it seems to hit in precise pinpoint flashes.

### **The Double-Slit Experiment — Fundamental Duality**

Something strange happens when both slits are open. The light still comes in pinpoint flashes, but the pattern it makes is not a simple combination of the patterns for the top and bottom slits. The same light that produces tiny flashes also seems to detect whether one or two (widely separated) slits are open. The fundamental question here is, “How does the light know to spread out so it can check how many slits are open in the first screen, while also knowing to make a tiny pinpoint flash on the second screen?”

## **3.3 Conclusions**

### **3.3.1 Additional Background**

Compelling as it would be for light alone to behave so, this peculiar effect (called nonlocality) is seen with other objects as well. Photons, electrons, even buckyballs show interference in double-slit experiments. Properly scaled, the graphs in the right column would look the same for experiments using any of these objects; all of these objects exhibit nonlocality. Nonlocal effects are the basis of quantum cryptography and quantum computing; only the nonlocal aspects of quantum

mechanics conflict with relativity.

### 3.3.2 Summary

The preceding chapter described the measurement problem, the lack of explanation for why a photon seems to go through both slits and produce a tiny pinpoint flash when it strikes the detecting screen. The following chapter will detail several attempts to solve this problem, some by introducing new physics and some by more carefully analyzing existing content.

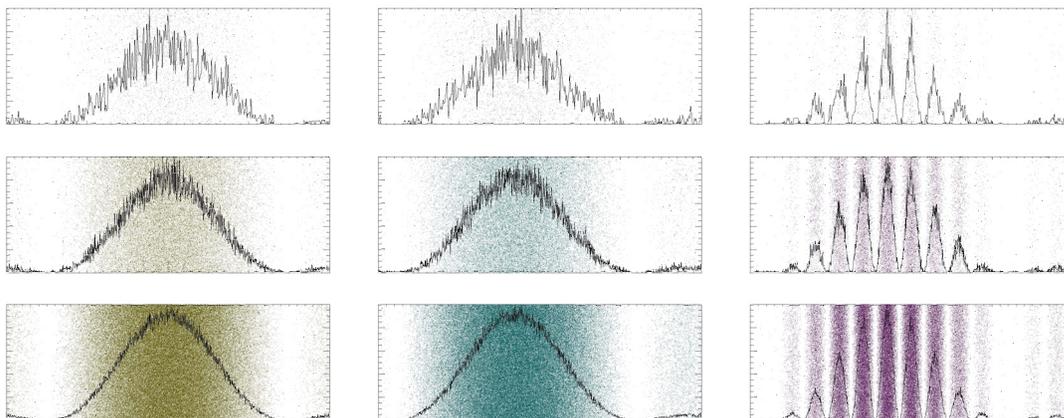


Figure 3.1: **Simulated Results from a Double-Slit Experiment:** These nine plots, made with code from [Boc05b], show the results from a simulated double-slit experiment. The first column shows the results with only the bottom slit open, the middle column with the top slit open, and the last column with both. If the particles used in the experiment behaved like bullets, the right column would show twin peaks, but as it does not, they are said to interfere with each other. Interference is most easily visible in the rightmost column; two-path experiments show this property extremely clearly. The top row shows a run for six seconds; the middle row shows a run for a minute; the bottom row shows a five-minute run.



# Chapter 4

## Solutions to Quantum Problems

### 4.1 Introduction

The following sections introduce various proposals to explain measurement.

### 4.2 Orthodox Theory

Nature at the quantum level is not a machine that goes its inexorable way. Instead what answer we get depends on the question we put, the experiment we arrange, the registering device we choose. We are inescapably involving in bringing about that which appears to be happening.

J.A. Wheeler, [Whe84]

Wheeler's sentiment reflects a common view of the strangeness of quantum mechanics, one at odds with a strictly physical viewpoint. Quantum mechanics does provide a machine, the Schrödinger equation, which goes on its inexorable way as long as we let it. Measurements seem to interfere with the functioning of this machine.

The orthodox theory, an ensemble average of the opinions of individual physicists, is a murky gallimaufry of vague and dubiously necessary definitions. Its treatment of measurement, following von Neumann, runs as follows.

### 4.2.1 Measurement in the Orthodox Theory

Consider a system to be measured and an observer. The state of the system is defined by a ray in a Hilbert space; let it be normalized. This state is defined as  $|\psi\rangle$ . The state of the system will evolve according to the Schrödinger equation until a measuring apparatus acts on it. The measuring apparatus is associated with a Hermitian operator  $\hat{O}$  defined on the Hilbert space containing the state of the system. The system in state  $|\psi\rangle$  can be written in the basis given by the normalized eigenvectors  $|o_k\rangle$  of the operator  $\hat{O}$  associated with the measuring apparatus as follows:

$$|\psi\rangle = \sum_k c_k |o_k\rangle \quad (4.1)$$

The action of the measuring apparatus on the system is random; upon measurement, the measuring apparatus causes the system to evolve from the state  $|\psi\rangle$  into one of the eigenvectors  $|o_k\rangle$  with probability  $c_k^*c_k$ . The normalization requirements for the state and eigenvectors ensure that these probability factors are also normalized.

### 4.2.2 Criticisms of Orthodox Measurement

The major criticisms of the above treatment of measurement stem from the conflict between the dynamics of measuring devices with the dynamics of non-measuring devices. If, in this theory, an observer, let us call it II, treats another observer as

part of a system being measured, observer II will have no way to choose whether to treat the observer being measured as an observer or as a quantum system, whether to expect the observer to display Schrödinger dynamics or the proposed non-unitary dynamics of measurement, called collapse.

A secondary concern lies with entangled spatially separated states; the collapse process, while it cannot be used to transmit chosen information, seems to be a form of action-at-a-distance. This seems to conflict with relativity.

### 4.2.3 Conclusions

A talented physicist can always draw the line between observer and system in a way consistent with experiment. For this reason, von Neumann's theory of measurement has never failed to explain an experiment. Nonetheless, it is unsatisfactorily arbitrary, adds unnecessarily to the theory, and seems to say less than it could about the dynamics of measurement. The following section details a proposal that addresses the third of these concerns.

## 4.3 Ghirardi-Rimini-Weber

In a 1986 paper [GRW86] Ghirardi, Rimini, and Weber propose a dynamical<sup>1</sup> solution to the Measurement Problem. Their exposition of the state of affairs:

“Almost all the difficulties [of Quantum Mechanics] can be traced back to the problem of accounting for the behavior of macroscopic objects and for their interactions with microscopic ones, and are strictly related to the occurrence (allowed by the theory) of linear superpositions of macroscopically distinguishable states of a macroscopic system

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<sup>1</sup>A modification of the dynamical equations of motion

[Schrödinger’s Cat states.]”

In order to prevent S-Cat states, GRW propose replacing the unitary<sup>2</sup> time evolution of QM with a stochastic equation. This approach succeeds at their stated goal.

### 4.3.1 Dynamics

GRW replace the quantum dynamical equation (equation 2.1 in [GRW86])

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar} [\hat{H}, \rho(t)] \quad (4.2)$$

with a modified version (equation 3.5 in [GRW86])

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar} [\hat{H}, \rho(t)] - \frac{\gamma}{4} [\hat{q}, [\hat{q}, \rho(t)]] . \quad (4.3)$$

### 4.3.2 Motivation

To derive equation 4.3, GRW start with equation 4.2 and consider adding a stochastic element to the dynamics. They want to transform linear superpositions of macroscopically distinguishable states into statistical mixtures of macroscopically distinguishable states. To do this they continually replace a small fraction ( $\lambda dt \ll 1$ ) of  $\rho(t)$ , which may or may not be a pure state, with that small fraction converted into a statistical mixture of position states. Mathematically, this operation can be written as

$$\frac{d}{dt}\rho(t) = -\lambda\rho(t) + \lambda T[\rho(t)] \quad (4.4)$$

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<sup>2</sup>Therefore deterministic, linear, and reversible

Where  $T[\rho]$  is  $\rho$  converted into a statistical mixture of position-space eigenstates.

In the Dirac language, if  $\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i|$  then

$$T[\rho] = \sum_x |\psi_x\rangle \langle\psi_x| \left( \sum_i p_i |\psi_i\rangle \langle\psi_i| \right) |\psi_x\rangle \langle\psi_x| \quad (4.5)$$

### Relation to Measurement

Define  $C_{ix} = \langle\psi_i|\psi_x\rangle$ . According to the Born probability rule, the square of this quantity,  $C_{ix}^* C_{ix}$ , is the probability that a system in the state  $|\psi_i\rangle$  will be found in the state  $|\psi_x\rangle$ . The action of a measurement of an observable with basis vectors  $|\psi_i\rangle$  on state  $|\psi\rangle$  is written in the orthodox manner as taking the pure state  $|\psi\rangle$  with density matrix  $\rho_\psi$

$$|\psi\rangle = \sum_i C_i |\psi_i\rangle \quad (4.6)$$

$$\rho_\psi = |\psi\rangle \langle\psi| \quad (4.7)$$

$$= \left( \sum_i C_i |\psi_i\rangle \right) \left( \sum_i C_i^* \langle\psi_i| \right) \quad (4.8)$$

$$= \sum_{i,j} C_i C_j^* |\psi_i\rangle \langle\psi_j| \quad (4.9)$$

to a mixed state with post-measurement density matrix  $\rho_m$

$$\rho_m = \sum_i C_i C_i^* |\psi_i\rangle \langle\psi_i| \quad (4.10)$$

$$\rho_m = \sum_i |C_i|^2 |\psi_i\rangle \langle\psi_i| \quad (4.11)$$

From this, Equation 4.5 can be rewritten in the form of Equation 4.11, defining

a few variables:

$$C_x = \sum_i \sqrt{p_i} \langle \psi_x | \psi_i \rangle \quad (4.12)$$

$$T[\rho] = \sum_x |C_x|^2 |\psi_x\rangle \langle \psi_x| \quad (4.13)$$

### Conclusion and Interpretation

Thus the effect of Equation 4.4 is verified. To derive Equation 4.3, which has no  $\lambda$ , GRW define the  $|\psi_x\rangle$  states as Gaussians with a small position-space width  $\frac{1}{\sqrt{\alpha}}$ . Taking the limit as  $\lambda \rightarrow \infty$  with  $\lambda\alpha = \gamma$ , Equation 4.4 produces the double-commutator term on the right of Equation 4.3.

However, the effect of Equation 4.3 has not been verified. The net effect of continuously converting small fractions of  $\rho_\psi$ , a pure state, to a statistical mixture is not to eventually replace  $\rho_\psi$  by a position-space statistical mixture  $T[\rho_\psi]$ . It is to select out a single position-space component of  $\rho_\psi$ , significantly reducing the amplitude for all other components. The one selected position-space component is very likely to be the one with the greatest initial value; this is called the “Gambler’s Ruin.” The position-space components not selected take infinite time to decay to zero amplitude. They are called “GRW tails” and can, with vanishingly small probability, grow to large amplitude and significantly affect the dynamics.

### 4.3.3 Criticisms

The GRW theory, one of several falling into the category of “Spontaneous Localization” theories, has been strongly criticized on both physical and philosophical grounds.

### Philosophically Motivated Criticisms

Cordero, in a philosophical defense of one feature of GRW [Cor99], divides the criticisms into four distinct claims.

1. “GRW’s relaxation of the Standard Orthonormal Rule (SOR<sup>3</sup>) leads to [significant interpretational problems.]”
2. “GRW cannot account for the determinate locations of macroscopic objects.”
3. The natural test of localization is having a position-space wavefunction that is nonzero only near some location.
4. The above “difficulties compromise standard arithmetic.”

Cordero responds effectively to these four criticisms.

The first he simply dismisses; relaxing the SOR causes states to “lack SOR-interpretation,” but this “does not entail that [they] defy interpretation.” As for the second, although GRW does not provide perfect localization in finite time, it provides localization good for all practical purposes. Cordero rejects the third by noting that such a wavefunction is physically impossible, yet we observe localized objects; therefore, this fails as a test of localization. Having answered the first three criticisms, the fourth fails.

### Physically Motivated Criticisms

Joos, in a comment [Joo87] on [GRW86], states that decoherence provides the following density-matrix equation for a free particle interacting with its natural en-

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<sup>3</sup>From context, this appears to be the definition that having a property is defined as being a linear multiple of that property’s basis vector

vironment:

$$\frac{\partial}{\partial t} \rho(x, x', t) = \frac{1}{2mi} \left( \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial x'^2} \right) \rho - \Lambda (x - x')^2 \rho. \quad (4.14)$$

Auletta [Aul01] cites this and presents it in the following form as his Equation 21.7.

$$\frac{\partial}{\partial t} \hat{\rho} = \frac{1}{i\hbar} [\hat{H}, \hat{\rho}] - \eta [\hat{q}, [\hat{q}, \hat{\rho}]] \quad (4.15)$$

Note the extreme similarity between 4.3 and 4.15. This shows that the effect of an environment on a system is indistinguishable from that of GRW's modified dynamics. This, combined with the *ad hoc* nature of the GRW proposal, seems adequate evidence to believe in decoherence rather than GRW. Although they do not conflict, the GRW theory provides no new physics.

## 4.4 Hidden Variables

“Physicists have only probability laws because for two generations we have been trained not to believe in causes – and so we have stopped looking for them.” – [Jay85]<sup>4</sup>

Quantum Mechanics is a stochastic theory; it does not tell us where a photon shall fall upon a detecting screen, or which path a neutron may take in an interferometer. Even to say “it does not tell us . . . which path” is misleading, for neutrons do not intrinsically have paths in QM.

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<sup>4</sup>E.T. Jaynes was a brilliant scientist. He understood probability better than anyone, but here his correctness is debatable. The theories of de Broglie, Bohm, and others attempt to provide causes for seemingly unpredictable quantum effects, and many would consider de Broglie and Bohm to be physicists.

To Jaynes, the natural response to an experiment with an uncertain outcome is to seek causes. Causes can be represented by supplementary parameters describing a physical system. As all theories that seek to assign supplementary parameters provide no way to measure or interact with these parameters, these theories are often called “hidden variables” (HV) theories.

In this section I shall present Bohm’s theory and some general criticisms of all HV theories.

#### 4.4.1 Dynamics of Bohm’s Hidden Variables Interpretation

To develop his Interpretation, Bohm takes Schrödinger’s Equation, an equation for the complex-valued wavefunction  $\psi$ , and writes it as two real-valued wave equations. This sets the stage for his introduction of hidden variables. The following is from his seminal paper [Boh52a].

$$\text{Schrödinger equation:} \quad i\hbar \frac{\partial \psi}{\partial t} = - \left( \frac{\hbar^2}{2m} \right) \nabla^2 \psi + V(x)\psi \quad (4.16)$$

$$\text{Complex Polar Wavefunction:} \quad \psi = R e^{iS/\hbar} \quad (4.17)$$

Bohm solves the Schrödinger Equation for R and S:

$$\frac{\partial R}{\partial t} = - \frac{1}{2m} [R \nabla^2 S + 2 \nabla R \cdot \nabla S] \quad (4.18)$$

$$\frac{\partial S}{\partial t} = - \left[ \frac{(\nabla S)^2}{2m} + V(x) - \frac{\hbar^2}{2m} \frac{\nabla^2 R}{R} \right] \quad (4.19)$$

Bohm identifies the magnitude of the real part of the wavefunction  $\psi$  as the square root of the probability density P and writes these equations in terms of S and P:

$$\frac{\partial P}{\partial t} + \nabla \cdot \left( P \frac{\nabla S}{m} \right) = 0 \quad (4.20)$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(x) - \frac{\hbar^2}{4m} \left[ \frac{\nabla^2 P}{P} - \frac{1}{2} \frac{(\nabla P)^2}{P^2} \right] = 0 \quad (4.21)$$

Bohm notes that in the classical limit ( $\hbar \rightarrow 0$ ) Equation 4.21 becomes the classical Hamilton-Jacobi Equation,

$$\text{General form: } \frac{\partial S}{\partial t} + H \left( q_1, \dots, q_n; \frac{\partial S}{\partial q_1}, \dots, \frac{\partial S}{\partial q_n} \right) = 0 \quad (4.22)$$

$$\text{Classical limit of 4.21: } \frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(x) = 0. \quad (4.23)$$

Therefore, in the classical limit the phase  $S$  of the wavefunction  $\psi$  is a solution to the Hamilton-Jacobi equation. Bohm then shows “that it is consistent to regard  $P(x)$  as the probability density for particles” in some ensemble, and that if this ensemble is chosen, “we can regard  $[P(x)\nabla S(x)/m]$  as the mean current of particles.” Bohm asserts that this interpretation is valid “even when  $\hbar \neq 0$ .”

#### 4.4.2 Introducing the Hidden Variables

By regarding  $P(x)$  as a probability density, Bohm has merely rewritten Schrödinger’s equation. However, Bohm asserts that  $P(x)$  is some kind of ensemble average and investigates the individual systems which compose this ensemble. Regarding  $P(x)$  as an average of  $n$  particles with initial position given by  $q_n$ , Equation 4.20 can be rewritten as the “guidance equation”

$$\frac{dq_i}{dt} = \frac{\nabla S(q_i)}{m}. \quad (4.24)$$

Bohm has given a particle in a quantum system a unique position. Given an ensemble of such particles with initial positions distributed according to  $\psi^*(x, t_0)\psi(x, t_0)$ , the future probability distribution for position space will be given by  $\psi^*(x, t)\psi(x, t)$ . Bohm's interpretation provides the same probability distributions for dynamics and measurement as the indeterministic orthodox interpretation, while assigning definite positions for particles at all times.

### 4.4.3 But What About von Neumann's Proof?

Bohm summarizes an important proof<sup>5</sup> on page 187 of [Boh52b] as stating "that no single distribution of hidden parameters could be consistent with the results of the quantum theory." This proof is completely correct, but Bohm's theory does not specify a single distribution of hidden parameters. According to Bohm, "when we measure the position "observable," the final result is determined in part by hidden parameters in the position-measuring device."

Bohm's statement reveals a crucial aspect of his theory: the results of measurements depend fundamentally on the measuring apparatus. Although Bohm's dynamics predict, for example, a definite value upon measurement of each of  $S_z$ ,  $S_x$ , and  $S_y$ <sup>6</sup> for some electron with initial position specified, Bohmian mechanics' prediction of the result of the  $S_z$  measurement depends on the arbitrary choice of the  $x$ - and  $y$ -axes. This property, that the outcome of an experiment depends on which other observables are selected as measurable by some apparatus, is called contextuality. Contextuality is often dismissed by Bohmians with the assertion that it ought to be obvious that the result of an experiment depends on an experi-

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<sup>5</sup>von Neumann's proof limiting the class of Hidden Variables theories consistent with Quantum Mechanics, the original on pages 167-171 of [vN32]

<sup>6</sup>The quantum theory makes probabilistic predictions for measurement of the spin component along any axis of an electron in an arbitrary state; Bohm's theory makes a deterministic prediction based on the hidden variables describing the electron and measurement apparatus

ment. There are some cases where this is far from obvious.

#### 4.4.4 Relativity and the Quantum Potential

That the contextuality of Bohm's Hidden Variables Interpretation is no simple dependence on experiment is particularly clear in its account of an Einstein-Podolsky-Rosen-Bohm<sup>7</sup> (EPRB) experiment.

The EPRB experiment explores the relationship between Relativity and Quantum Mechanics. One prediction of Relativity is that physical influences cannot propagate faster than light. The effects that follow from some cause are contained within the forward light cone of that cause. This is called locality.

Quantum Mechanics predicts correlation between spin measurements on a pair of particles in a spin-singlet state, regardless of the separation at time of measurement. Many physicists believe that it is possible to generate two sets of random numbers with no correlation, so that arbitrary measurements can be made with no common cause. Then wave-function collapse is triggered by one or another arbitrary measurement, so there should not be causal influences between the two measurements, but there must be. QM avoids this by making statistical predictions, but even these are non-local. Bohm's theory requires deterministic faster-than-light effects, although in the quantum equilibrium they cannot be used for signaling.

### 4.5 Everett's Relative State Formulation

I do not group my discussion of Everett's original paper [HE57] with that of the Many-Worlds Interpretation out of respect for the attention it drew to the physics

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<sup>7</sup>See [Boh51] pages 614-23.

of measurement. Although Everett produces neither a new footing nor a new probability structure for quantum mechanics, his discussion of relative states, physical devices as observers<sup>8</sup>, and repeated measurements moves toward clarity. Auletta, on page 247 of [Aul01] states that Everett's formulation "gave a clue toward Zurek's solution [decoherence]."

### 4.5.1 Motivation

Everett begins by noting the two "fundamentally different" and incompatible kinds of time evolution in Quantum Mechanics, the "discontinuous change brought about by ... observation" and the linear dynamics of the Schrödinger equation. Everett then asserts that Quantum Mechanics with only the second (linear) kind of time evolution forms a complete theory, although it's not clear what this assertion entails. As Everett does not know of decoherence, there are clear flaws in his theory.

### 4.5.2 Mathematics

Everett's main point in [HE57] is that interacting systems become entangled during measurement, and that the state of the entangled system can be written as what he calls a "relative state." For clarity I use the conventions of [Aul01].

### 4.5.3 Observation

Everett offers a simple formula prescription for writing down the process of a good measurement:

Given a system  $\psi^{S_1+O}$  with wavefunction  $\psi^{S_1}\psi^O$  it can be written by the discussion of the previous section as  $\sum_i |i^{S_1}\rangle \langle i^{S_1}|\psi^{S_1}\psi^O\rangle$ . Everett defines the inner

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<sup>8</sup>That is, capable of inducing collapse

product  $\langle i^{S_1} | \psi^{S_1} \psi^O \rangle$  as  $\alpha_i |O_{[...]} \rangle$ , where [...] is the memory of the observing system. Everett defines the action of measurement as replacing the above state with a post-measurement state  $\sum_i \alpha_i |i^{S_1} \rangle |O_{...,i} \rangle$ . The observer in the  $i^{\text{th}}$  element of the sum records having measured the  $i^{\text{th}}$  eigenvalue.

Unfortunately this suffers from what is called the “basis degeneracy” problem. First I’ll present a simple mathematical example, then discussion.

### Example

Consider a spin- $\frac{1}{2}$  qubit in the computational basis with definite z-spin up. In the Dirac language this is written  $|0\rangle$ . An alternate basis for such qubits is the x-spin basis. Its basis kets are

$$|\uparrow_x\rangle = \sqrt{\frac{1}{2}} |0\rangle + \sqrt{\frac{1}{2}} |1\rangle \quad (4.25)$$

$$|\downarrow_x\rangle = \sqrt{\frac{1}{2}} |0\rangle - \sqrt{\frac{1}{2}} |1\rangle \quad (4.26)$$

Let us direct our apparatus to perform a set of measurements on this system. In our apparatus there is a digital computer with a memory; it will record the results of a series of three measurements: z-spin, x-spin, and then z-spin again. Before the first experiment, the state of the system and apparatus together can be written:

$$|\psi_0^{S+A}\rangle = \sum_i |i\rangle |O_{[...]} \rangle \quad (4.27)$$

$$= |0\rangle |O_{[...]} \rangle \quad (4.28)$$

Here [...] denotes a blank memory and  $|i\rangle$  denotes one of the kets which make up

the computational basis. After the first measurement, according to Everett:

$$|\psi_1^{S+A}\rangle = |0\rangle |O_{[0]}\rangle \quad (4.29)$$

The second measurement is slightly more interesting.

$$|\psi_1^{S+A}\rangle = \sum_i |x_i\rangle \langle x_i|0\rangle |O_{[0]}\rangle \quad (4.30)$$

$$= \sqrt{\frac{1}{2}} |\uparrow_x\rangle |O_{[0]}\rangle + \sqrt{\frac{1}{2}} |\downarrow_x\rangle |O_{[0]}\rangle \quad (4.31)$$

$$|\psi_2^{S+A}\rangle = \sqrt{\frac{1}{2}} |\uparrow_x\rangle |O_{[0,\uparrow_x]}\rangle + \sqrt{\frac{1}{2}} |\downarrow_x\rangle |O_{[0,\downarrow_x]}\rangle \quad (4.32)$$

The third measurement shows real problems.

$$|\psi_2^{S+A}\rangle = \sum_i |i\rangle \langle i| \left( \sqrt{\frac{1}{2}} |\uparrow_x\rangle |O_{[0,\uparrow_x]}\rangle + \sqrt{\frac{1}{2}} |\downarrow_x\rangle |O_{[0,\downarrow_x]}\rangle \right) \quad (4.33)$$

$$= |0\rangle \langle 0| \left( \sqrt{\frac{1}{2}} |\uparrow_x\rangle |O_{[0,\uparrow_x]}\rangle + \sqrt{\frac{1}{2}} |\downarrow_x\rangle |O_{[0,\downarrow_x]}\rangle \right) \quad (4.34)$$

$$+ |1\rangle \langle 1| \left( \sqrt{\frac{1}{2}} |\uparrow_x\rangle |O_{[0,\uparrow_x]}\rangle + \sqrt{\frac{1}{2}} |\downarrow_x\rangle |O_{[0,\downarrow_x]}\rangle \right) \quad (4.35)$$

$$= \frac{1}{2} |0\rangle |O_{[0,\uparrow_x]}\rangle + \frac{1}{2} |0\rangle |O_{[0,\downarrow_x]}\rangle \quad (4.36)$$

$$+ \frac{1}{2} |1\rangle |O_{[0,\uparrow_x]}\rangle - \frac{1}{2} |1\rangle |O_{[0,\downarrow_x]}\rangle \quad (4.37)$$

$$= \frac{1}{2} |0\rangle |O_{[0,\uparrow_x+\downarrow_x]}\rangle + \frac{1}{2} |1\rangle |O_{[0,\uparrow_x-\downarrow_x]}\rangle \quad (4.38)$$

$$|\psi_3^{S+A}\rangle = \frac{1}{2} |0\rangle |O_{[0,\uparrow_x+\downarrow_x,0]}\rangle + \frac{1}{2} |1\rangle |O_{[0,\uparrow_x-\downarrow_x,1]}\rangle \quad (4.39)$$

$$|\psi_3^{S+A}\rangle = |0\rangle |O_{[0,0,0]}\rangle \quad (4.40)$$

The observer system finds that its result for the second measurement is a superposition state; if we have it ask itself after the series of three measurements, “does the result for the second measurement possess a definite z-spin value?,” it

will answer “yes,” because our measuring apparatus is defined to yield definite results. But because the measuring device firstly does not change the state of the system being measured, and secondly measures states faithfully, the third measurement must match the first. Also, by the linearity of quantum mechanics, the superposition of the observer’s second memory term in Equation 4.39 must simplify in the computational basis to the result of Equation 4.40. So the final state of the system is incompatible with that predicted by collapse,

$$\hat{\rho} = \frac{1}{4} |O_{[0,\uparrow_x,0]}\rangle \langle O_{[0,\uparrow_x,0]}| + \frac{1}{4} |O_{[0,\uparrow_x,1]}\rangle \langle O_{[0,\uparrow_x,1]}| \quad (4.41)$$

$$+ \frac{1}{4} |O_{[0,\downarrow_x,0]}\rangle \langle O_{[0,\downarrow_x,0]}| + \frac{1}{4} |O_{[0,\downarrow_x,1]}\rangle \langle O_{[0,\downarrow_x,1]}| \quad (4.42)$$

#### 4.5.4 Conclusions

Everett’s Relative State Formulation lacks two things; the addition of these two things transforms it into the Many-Worlds Interpretation. They are: a way of determining when a measurement has occurred and a way of determining what has been measured.

W.H. Zurek, in [Zur81], demonstrates that “quantum mechanics alone . . . cannot in principle determine which observable has been measured.” This is dictated by the linearity of QM. Given that QM cannot determine which observable has been measured, the potential remains to measure any observable, and so QM cannot determine that a measurement has occurred, although it can describe much of the interaction between a measuring apparatus and a system to be measured. Everett, with his relative states, aids in this description, but as he does not add to the formalism of Quantum Mechanics, his theory provides no solution to the problems of measurement.

### 4.5.5 The Many Worlds Interpretation “and other bizarre theories”

Despite (before, actually) Zurek's demonstration, De Witt<sup>9</sup>. [DeW70] claims that Everett's belief in the completeness of wave mechanics “forces us to believe in the reality of all the simultaneous worlds represented in the superposition<sup>10</sup>.”

The fundamental counter-argument here is quite simple. We return to the Double Slit Experiment. If a certain small device is placed in front of the first screen, one can either measure which slit the photon went through (destroying the interference pattern) or choose not to disturb the photon (preserving interference). This choice can be made after the photon has passed through the first screen and is flying towards the second. The Many-Worlds Interpretation asserts that different relative states correspond to different worlds, and that worlds split upon measurement. Here, however, the observer has a choice about when to perform a measurement, a choice that only subtly affects the physical situation. This ambiguity as to what constitutes a measurement is the fundamental weakness of the Many-Worlds Interpretation, and without a definition of measurement, the MWI explains nothing.

There is a second problem, that the splitting of universes is asymmetric in time. The ambiguity in measurement and unnecessary denial of time-reversibility fatally wound this theory.

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<sup>9</sup>Peres heads a section on page 374 of his textbook [Per95] with the text “Everett's interpretation and other bizarre theories,” it seems here appropriate as well.

<sup>10</sup>Page 161, [DeW70]

## 4.6 Decoherence

Each of the above interpretations has added an ingredient of its own to the quantum theory; the results were satisfactory to their authors, but none completely fit into the theory. GRW changes the dynamics, Hidden Variables adds its titular concept, and the Many Worlds Interpretation claims to add nothing, but sneaks in the old ideas of measurement.

The decoherence program begins with the observation that quantum experiments take place in a laboratory as much as they do a Hilbert space. The practicalities of real experiments reveal overlooked structures in the process of measurement. Although “delayed-choice” experiments are possible, they require careful experimental design to shield the quantum system from the (necessarily noisy) interaction with the environment. Without this care, the choice cannot be delayed, and will be made for us by the experimental apparatus. Quantum decoherence is an effect of the environment (the laboratory, outside world, and disregarded degrees of freedom in the quantum system itself) on quantum systems. I consider its effect on quantum measurements, so the system undergoing decoherence will here be the combination of a system to be observed and an associated measuring apparatus.

### 4.6.1 Aside: Quantum Zeno Effect

The interaction with the environment that produces decoherence can also produce two related effects. One of them is the Quantum Zeno Effect.

Consider the orthodox view of projective measurements. Consider a two-state system with initial state  $|0\rangle$  and a Hamiltonian that causes this state to oscillate to  $|1\rangle$  and back over some time constant  $2\tau$ . Suppose now that the system is mea-

sured at two times:  $t = 0$  and  $t = \tau$ . The first measurement will show the system to be in state  $|0\rangle$  with certainty and the second will show it to be in state  $|1\rangle$  with certainty. Suppose now that a third measurement is taken between the other two, at time  $t = \tau/2$ . The system, according to the orthodox view, will collapse into an even mixture of the two states; each will evolve until the final measurement at  $\tau$ . Each component of the mixture has an even chance of either switching states or not, leaving an even mixture at  $\tau$ . This effect can be demonstrated in experiments.

The results of the above measurements on the above two-state system are often interpreted in the following way. With observation only at  $t = 0$  and  $t = \tau$ , the system seems to evolve from state  $|0\rangle$  to state  $|1\rangle$  in  $\tau$  time intervals. Performing a third measurement at  $t = \tau/2$  leaves the system in an even mixture at  $t = \tau$ . Some describe this state of affairs thusly: "Because the system with two measurements was certain to flip from 0 to 1, and the system with three measurements had only probability .5 to flip from 0 to 1, time evolution has slowed down in the triply-measured system." With a fourth and fifth measurement at  $t = \tau/4$  and  $t = 3\tau/4$ , the final state will be  $5/8$  in  $|0\rangle$  and  $3/8$  in  $|1\rangle$ . Increasing numbers of equally spaced measurements yield still greater probability that the system will be measured to be in state  $|0\rangle$  at  $t = \tau$ . See Figure 4.1. This is often summarized as "Increasing the number of measurements throughout the interval will cause the dynamical evolution of the system to slow down." This is called the Quantum Zeno or Quantum Watched Pot effect; a watched pot never boils. In practice, the approximation of instantaneous perfect measurements breaks down. The following example exhibits both decoherence and the quantum Zeno effect.

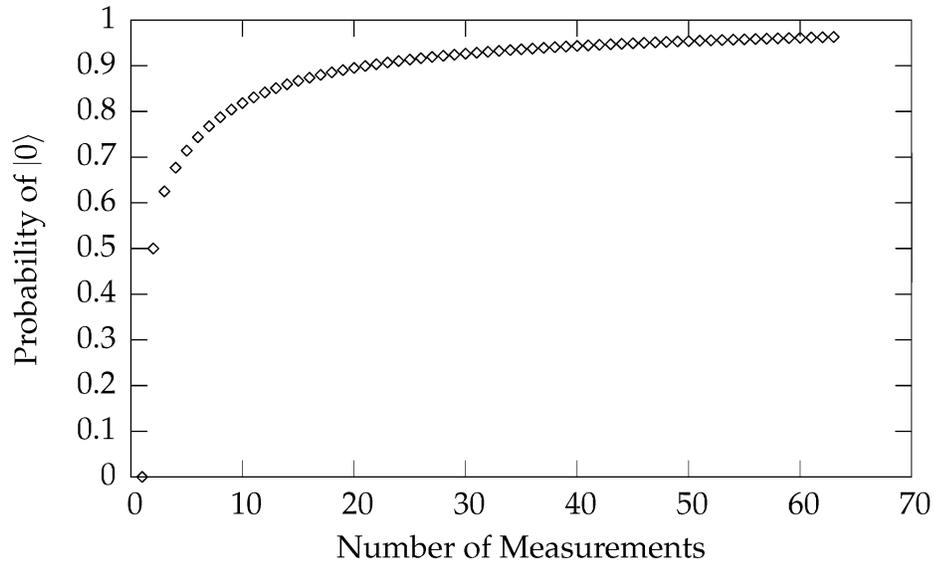


Figure 4.1: This figure shows the probability of finding the two-state system discussed in subsection 4.6.1 still in the  $|0\rangle$  state after a time interval of length  $\tau$ , with  $N$  evenly spaced measurements occurring during this interval. Note that the more closely the system is observed, the more likely it will remain in the  $|0\rangle$  state.

## 4.6.2 Mathematical Example for a Two-State System

In [Ber95], Berry presents a model (from [HS81]) based on “the simplest non-simple quantum problem, namely, the evolution of two-state systems with a time-dependent Hamiltonian” which displays decoherence and other related environment-induced effects.

The state of the system in Berry’s model is represented by a 2-spinor

$$|\psi\rangle = \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} \quad (4.43)$$

As the global phase of the system can be disregarded, we can write the Hamiltonian, and therefore the Schrödinger equation as a dot product of a 3-vector and the vector of Pauli matrices  $\mathbf{S}$ . If the global phase mattered, the dot product of a

4-vector and the set of three Pauli matrices and the 2x2 identity matrix would be required.

$$i\partial_t |\psi\rangle = \mathcal{H}(\epsilon, t) |\psi\rangle \quad (4.44)$$

$$\mathcal{H}(\epsilon, t) = \mathbf{R}(\epsilon, t) \cdot \mathbf{S} \quad (4.45)$$

$$i\partial_t |\psi\rangle = (\mathbf{R}(\epsilon, t) \cdot \mathbf{S}) |\psi\rangle \quad (4.46)$$

$$\mathbf{S} = \left(\frac{1}{2}\right) \left[ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \quad (4.47)$$

$$\mathbf{R}(\epsilon, t) = [X(\epsilon, t), Y(\epsilon, t), Z(\epsilon, t)] \quad (4.48)$$

Berry's model consists of a particle in one of two states (for him, "wells") separated by a high barrier. The energy eigenstates, superpositions of pairs of localized states, are separated by an energy  $\epsilon$ , exponentially small in the height and width of the barrier and  $1/\hbar$ . Systems initially localized in one of the wells oscillate ("tunnel resonantly") between wells with frequency proportional to  $\epsilon$ . The Hamiltonian for this (isolated) system is

$$\mathcal{H}_0(\epsilon, t) = \mathbf{R}_0 \cdot \mathbf{S} = \left(\frac{1}{2}\right) \begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix} \quad (4.49)$$

Berry's model for the interaction of the system with the environment is the "crude, but adequate" approximation of a time-dependent energy difference between the two localized states. This mimics statistically independent unpredictable fluctuations in the local potential energy for each of the two wells. To simplify analysis, he uses an energy difference distributed as white noise, although proves a more general result in an appendix to his paper. He introduces a white noise function  $f(t)$  normalized according to (with overbar denoting ensemble average-

ing)

$$\overline{\left(\int_0^T f(t) dt\right)^2} = \frac{T}{T_0} \quad (4.50)$$

He sets the z-component of  $\mathbf{R}$  equal to this, yielding a total time-dependent Hamiltonian describing both internal system dynamics and interaction with the environment:

$$\mathcal{H}(\epsilon, t) = \mathbf{R} \cdot \mathbf{S} = \left(\frac{1}{2}\right) \begin{pmatrix} f(t) & \epsilon \\ \epsilon & -f(t) \end{pmatrix}. \quad (4.51)$$

In order to interpret this result it is necessary to introduce a device called the Bloch vector for visualizing the state of a two-level quantum system. For a quantum system with a wavefunction that is a complex superposition of two basis vectors ( $a, b$  complex),

$$|\psi\rangle = a|0\rangle + b|1\rangle \quad (4.52)$$

there are four degrees of freedom. Neglecting the global phase of this state as it is not observable, the state can be written with  $c, d, \theta$  real:

$$|\psi\rangle = c|0\rangle + de^{i\theta}|1\rangle \quad (4.53)$$

Requiring normalization,  $\langle\psi|\psi\rangle = 1$ ,  $c^2 + d^2 = 1$ , reduces the number of degrees of freedom to two. Every state can be mapped to a point on the unit sphere, called the Bloch sphere when mentioned in the context of Bloch vectors. The mapping

used is

$$\vec{r}(t) = 2 \langle \psi(t) | \mathbf{S} | \psi(t) \rangle. \quad (4.54)$$

$$\vec{r}(t) = [2ab \cos \theta, -2ab \sin \theta, a^2 - b^2] \quad (4.55)$$

Berry calculates the time evolution of the ensemble-averaged density matrix and expresses it in terms of the ensemble-averaged Bloch vector  $\overline{\vec{r}(t)}$ .

$$\overline{\vec{r}(t)} = 2 \overline{\langle \psi(t) | \mathbf{S} | \psi(t) \rangle}. \quad (4.56)$$

$$\rho(t) = \overline{|\psi(t)\rangle \langle \psi(t)|} = \begin{pmatrix} \overline{|\psi_1(t)|^2} & \overline{\psi_1^*(t)\psi_2(t)} \\ \overline{\psi_2^*(t)\psi_1(t)} & \overline{|\psi_2(t)|^2} \end{pmatrix} \quad (4.57)$$

$$\equiv \left(\frac{1}{2}\right) + \mathbf{S} \cdot \vec{r}(t) = \left(\frac{1}{2}\right) \begin{pmatrix} 1 + z(t) & x(t) - iy(t) \\ x(t) + iy(t) & 1 - z(t) \end{pmatrix} \quad (4.58)$$

For a pure state,  $\vec{r}$  lies on the surface of the Bloch sphere; for a mixed state it lies in the interior. In the  $z$ -basis, which is here the “which well”-basis, the  $x$  and  $y$  components of the Bloch vector describe the off-diagonal components of the density matrix, as can be seen from equation 4.58. During measurement, these components go to zero; one process, perhaps the only process that causes this is decoherence. Berry determines the dependence of  $\vec{r}(t)$  on time and finds a solution in terms of a parameter  $u$  which he defines as

$$u \equiv 4\epsilon T_0 \quad (4.59)$$

This parameter is proportional to the quotient of the quantum oscillation frequency and the noise intensity. For the initial condition of  $\vec{r}(0) = \vec{r}_0$ , Berry deter-

mines the evolution to be

$$x(t) = \exp(-t/2T_0) \quad (4.60)$$

$$\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \frac{\exp(-\alpha_+ t)}{2\sqrt{1-u^2}} \begin{pmatrix} (1 + \sqrt{1-u^2}) y_0 + uz_0 \\ -uy_0 - (1 - \sqrt{1-u^2}) z_0 \end{pmatrix} \quad (4.61)$$

$$+ \frac{\exp(-\alpha_- t)}{2\sqrt{1-u^2}} \begin{pmatrix} -(1 - \sqrt{1-u^2}) y_0 - uz_0 \\ uy_0 + (1 + \sqrt{1-u^2}) z_0 \end{pmatrix}, \quad (4.62)$$

where

$$\alpha_{\pm} \equiv \left( \frac{1}{4T_0} \right) \left( 1 \pm \sqrt{1-u^2} \right) \quad (4.63)$$

For large  $u$ , the system exhibits quantum oscillations damped by tunneling friction. The system oscillates between the two wells, but decays slowly, after many oscillations, into a mixture. The small- $u$  limit concerns us here. From the limits

$$\alpha_+ \rightarrow \frac{1}{2T_0}, \quad \alpha_- \rightarrow 2\epsilon^2 T_0, \quad \text{as } u \rightarrow 0 \quad (4.64)$$

Berry arrives at the following picture of the density matrix flow. There is a line  $x = 0, y = -zu/(1 + \sqrt{1-u^2})$  which I shall call the decoherence line. On the scale of the noise time  $T_0$ , which is fast as compared with the quantum oscillation time  $1/\epsilon$ , the fast exponential, involving  $\alpha_+$ , causes the Bloch vector to flow from the surface of the sphere to the decoherence line. This is decoherence. Thereafter the slow exponential, involving  $\alpha_-$ , causes the Bloch vector to flow down the decoherence line towards the origin. This is the Quantum Zeno Effect.

### 4.6.3 Conclusions

It is clear that any model for decoherence is only able to quash the off-diagonal terms in the density matrix by averaging over some unknown parameter; without this, we would be left with Zurek's demonstration<sup>11</sup> that "quantum mechanics alone . . . cannot in principle determine which observable has been measured." By adding our knowledge of real measuring devices to quantum mechanics, we are able to determine which observable has been measured. Nonetheless, problems remain. Many are uncomfortable with this kind of averaging, having found what seem to be two different kind of probabilities in quantum mechanics. Regardless, decoherence is a demonstrable physical effect with interesting thermodynamic and engineering properties.

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<sup>11</sup>[Zur81]



# Chapter 5

## Conclusion

### 5.1 Einstein-Podolsky-Rosen: It didn't come up?

In a famous paper from the 1930s, Einstein, Podolsky, and Rosen suggested a simple set of definitions for physical theories and discussed an interesting aspect of QM. Bohm offered a more convenient experimental realization of the involved concepts. EPR saw complementarity in the position and momentum variables (operators (quantities (qualities))) of a particle. This same complementarity is present in the spin directions of a pair of photons in the spin-singlet state, but more easily considered and measured. Furthermore, the photons can be separated by a great distance, allowing relativistically space-like separation between measurements performed on the photons. This provides a feasible experimental test of nonlocal aspects of QM.

EPR also defined a few philosophical notions. Key among them was the idea of an “element of reality”, their term for a quantity with a definite value upon measurement. I shall refrain from discussing the implications of this here. EPR then stated that each “element of reality” must have a corresponding element in the theory. Theories that satisfy this condition are called complete.

In an EPRB experiment, two photons with opposite polarization are produced<sup>1</sup>. The apparatus can be arranged so that each photon has its polarization measured, and furthermore, that these two measurement events are separated by a space-like spacetime interval. This means that there exist reference frames in which either photon is measured first. The wavefunction collapse (WFC) then assigns upon measurement of the first photon a definite value for the spin of the other; this is a nonlocal effect. EPR claim that without collapse, QM lacks an element of physical reality corresponding to the definite value for the spin of the second photon.

I see three (two) prongs of attack: deny the value of identifying “elements of physical reality” with elements of the theory, deny the EPR-interpretation that nonlocal effects occur on wavefunction collapse, or note that Quantum Field Theory is less profligate in assigning definite values to measurements. The last prong is due to Daniele Tommasini [Tom03], who cleverly notes that a two-photon spin-singlet state cannot be created with certainty. This is no minor quibble. While it’s obvious that no detectors are perfect, so no measurement events are certain, that is a degenerate case: ideal detectors have a place in the theory. Tommasini identifies a fundamental quantum uncertainty in the EPRB state, before and after the first measurement; therefore, the putative “element of physical reality” required by a definite spin state need not exist. QFT rejects EPRB neither by denying locality nor by rejecting the EPRB criterion for realism, but by reducing the role of reality in a physical theory.

Still, the compatibility of quantum mechanics with relativity is a deep and open theoretical question, and I have entirely ignored issues of locality. The measurement theories I have presented have sufficient merits and flaws without drawing in concerns over spatially separated entangled states.

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<sup>1</sup>by parametric down-conversion. Some experiments use entangled energies rather than spins.

## 5.2 Decoherence and Reality

Decoherence explains why an external observer will find a quantum measurement system in a classically consistent state: uniquely quantum correlations become hidden in the degrees of freedom of the measurement apparatus. The Many-Worlds Interpretation claims to explain this, with its mechanism of splitting worlds, but delayed-choice experiments (effectively simple quantum computers) show that worlds do not split. These experiments imply that with a properly shielded laboratory, the choice of what to measure, and hence how to split the worlds, can be delayed for an arbitrary time.

However, the idea that the appropriate connection between probability and reality is the Born rule, with no collapse, appeals to me. What is lost is a consistent time evolution from past to future. After a no-collapse measurement, the observing apparatus is entangled with the observed system, and after a second measurement, it is very likely that the apparatus will have recorded identical values for the two measurements, but both outcomes remain possible. This situation can be viewed as “many decohering worlds:” worlds do not truly split, but seem to.

The last paragraph, however, is incorrect. If all quantum systems can be described as an ensemble of pure states, and the density matrix only represents some additional but not physically necessary uncertainty as to the state of a system, then the uncertainty as to what value was obtained upon measurement comes directly from the uncertainty in the initial conditions and subsequent dynamical evolution of the measuring device and apparatus. Given that a system begins in a pure state, it shall remain in a pure state. From this and a belief in the universality of quantum mechanics it follows that there is only one world, although observers do not know which of the possible worlds (those states  $|\psi\rangle$ ) consistent with their

memories) is the actual world.

But we would like an interpretation of measurement which works for incomplete knowledge of the system, for observers who deal with noisy apparatus in real laboratories. I think that accepting the probabilities given by the density matrices of decohering quantum systems provide this interpretation; however, no quantum theory can satisfy our every desire.

### 5.3 Peres and Zurek's Three Wishes

Our wishes for an interpretation of measurement echo those of Peres and Zurek in [PZ82]. They voice three (contradictory) wishes for a quantum theory: determinism, verifiability, and universality, presented in Figure 5.1. They analyze the consequences of pairs of these three wishes.

Those who seek a deterministic quantum theory must choose between verifiability and universality. Determinism and universality prevent freely chosen physical experiments, and such a theory cannot be tested. A verifiable deterministic theory is constrained by the theorems due to Bell and others limiting hidden variables theories.

Peres and Zurek choose verifiability and universality, the two wishes compatible with quantum theory, having clearly shown the relative merits of the three paths.

### 5.4 Some Concluding Thoughts

Quantum Mechanics, the branch of physics that describes the nature of reality on small scales, has shown unrivaled success at predicting and explaining experimental results since its inception at the beginning of the previous century. Despite

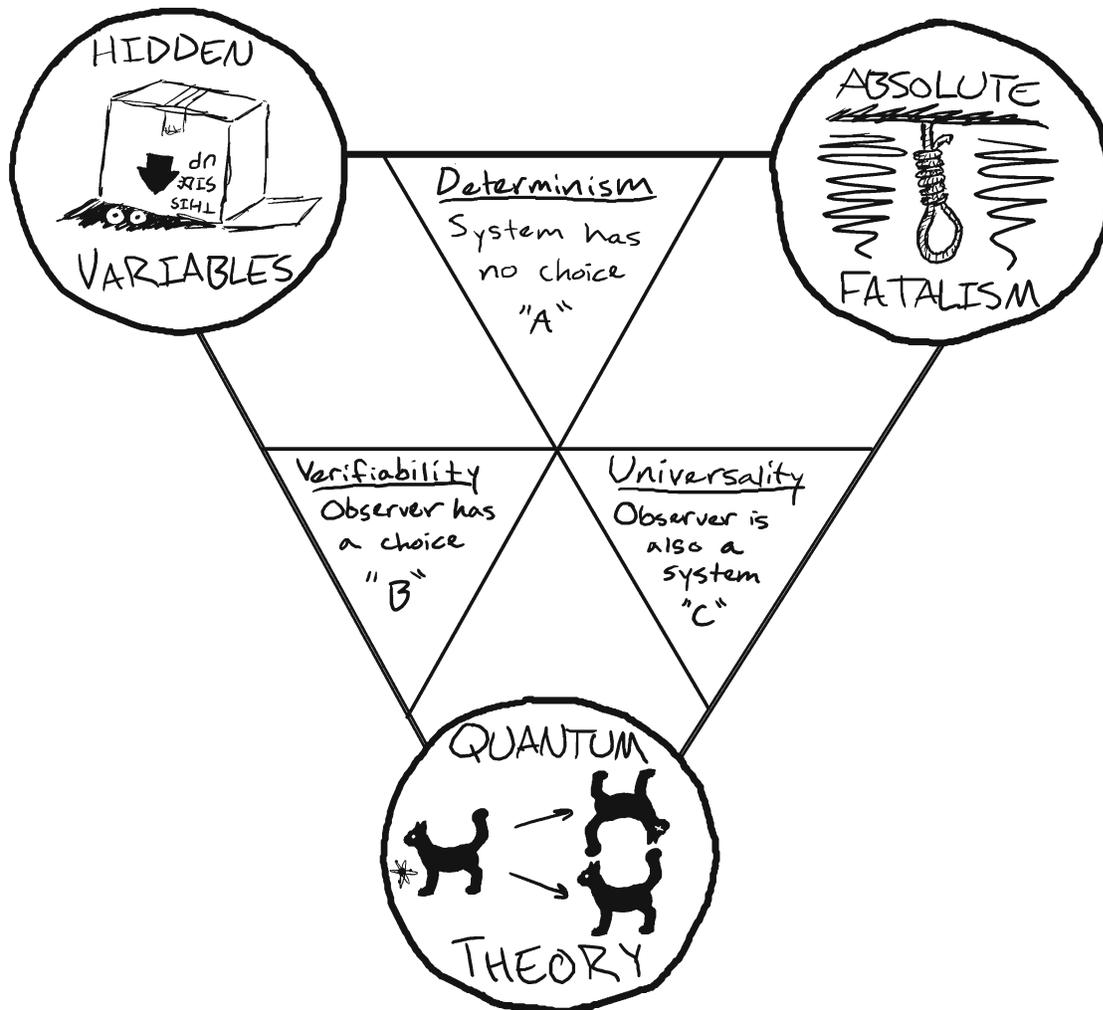


Figure 5.1: Three Wishes and their Consequences. The center holds three mutually contradictory wishes for a physical theory. As all three cannot be granted, we must choose two. The consequences of our choice are illustrated in the circles. This figure based on the original in [PZ82], was redrawn and illuminated by Eliza Blair[Bla06].

this success, debates continue over the physical content and proper interpretation of this theory, and for good reason: it is difficult to see how the rules of quantum mechanics create a world consistent with that we see. The quantum world is fuzzy, incomprehensibly small, and hard to see without a laboratory.

Fuzziness and incomprehensibility are no obstacle to determined physicists. These alone would not produce the mysteries of Quantum Mechanics. Real trouble comes from the ad hoc solution (called collapse) proposed at the beginning of Quantum Mechanics: the world was divided into classical objects like people and machinery that followed the classical rules and quantum objects like electrons that did not. This division was proposed despite the belief that people and machinery were composed of electrons and other quantum objects, and lead to no end of trouble.

The appropriate resolution to this ad hocery was conceptually simple: the difference between a person and an electron is one of scale. Persons require many electrons to display attributes of person-hood; so too does a speck of dust require many electrons for its composition. It turns out that a large collection of quantum objects acts very much like a classical object. The process that hides much quantum behavior of large collections of quantum objects is called decoherence. Understanding this process suggests new experiments that are unlikely to occur to those who believe in collapse.

In the course of my research, I encountered other proposed solutions, but each had more serious flaws than decoherence. Bohmian mechanics, an example of a hidden-variable theory, is nonlocal, whereas Quantum Mechanics is not fundamentally so. Mermin's Ithaca Interpretation seems to sacrifice too much physical content, although its claim that correlations have physical reality, whereas their correlata do not is compelling. Asher Peres, in his paper "Einstein, Podolsky,

Rosen, and Shannon," shows that collapse is consistent with relativity, but decoherence provides a physical explanation for collapse.

The primary finding of this research is that Quantum Mechanics, though better understood than ever before, still offers both subtle difficulties and hope for the future to its students. Modern experimental techniques have allowed thought experiments proposed in the Twenties and clarified thirty years later to be performed; research into quantum computing will surely advance our understanding of measurement further. This field has yielded quantum cryptography, only now beginning to be commercially applied, and offers advancement of both our applied and theoretical understanding of the world.



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